MATH1040 Assignment 3

All questions should be submitted by 6pm on Wednesday 21 March. You should show full working where possible. Assignments are to be submitted during your tutorial. Make sure that your name and student number are on each sheet of your answers. Solutions will be distributed in class later.

1. Answer each of the following questions, showing all working.

   (a) Simplify \( \frac{x^0 y^{-3}}{x^2 y^0} \)

   (b) Simplify \( x^1 y^2 - 1 \div (x^{-1} y^0) \times y^2 \)

   (c) Expand and simplify \( \sum_{i=-2}^{1} 3iz \)

   (d) Expand and simplify \( \sum_{i=2}^{5} (-1)^i i \)

   (e) Write in summation notation: \( \frac{-4}{5} + \frac{-4}{6} + \frac{-4}{7} + \frac{-4}{8} \)

   (f) Find \( x \) if \( \sum_{i=0}^{2} xi = 6 \)

2. Write the following in summation notation.

\[ 2 - 4x^4 + 3 - 9x^6 + 4 - 16x^8 + 5 - 25x^{10} + 6 - 36x^{12} \]

3. Felicity the Philatelist goes to the post office, to buy stamps. She buys a certain number of 50 cent stamps, and three times that many $1.10 stamps. All up, she spends $19.00. How many of each type of stamp does she buy?

4. Solve the equation \( 4x - 1 = \frac{1}{4}(x - 4) \) (Previous exam question.)

5. Simplify \( -e^2 \div e^3 \times \frac{1}{e} - e(e - e^{-3}) \)

6. For the following questions let \( A = \{ x \mid x \in \mathbb{N}, 1 \leq x < 4 \}, B = \{ 0, 1, 8, 10 \} \) and \( C = \{ -2, 0, 2, 4, 6, 8 \}. \)

   (a) Write down the elements of the set \( A \)

   (b) Write down the elements of the set \( A \cup B \)

   (c) Write down the elements of the set \( B \cap C \)

   (d) Write down the elements of the set \( A \setminus C \)

   (e) Write down the elements of the set \( A \setminus (B \cup C) \)

   (f) Write down the elements of the set \( (A \cap B) \cap C \)

(continued over...)
7. A recent American television soap opera has female characters Nora, Ophelia and Penelope. It also has male characters Alfred, Bruce, Cecil, Denzell, Engelbert, Frederick, Gilbert, Hank and Ignatius. Nora had affairs with Frederick, Denzell, Gilbert and Cecil. Ophelia had affairs with Bruce, Gilbert, Cecil and Alfred. Penelope had affairs with Engelbert, Frederick, Gilbert, Denzell and Bruce. When listing sets, just use the first letter of each man’s name.

(a) Define three sets $N$, $O$ and $P$, each giving the set of male characters with whom each female character has had an affair ('N' for Nora, and so on). List the elements of each set.

(b) Draw a Venn diagram to represent the tangled situation. Write the elements of each set on the Venn diagram, in the appropriate places. If any men do not appear in any set, write them outside the diagram.

(c) Convert each of the following statements to mathematics (for example, using $\cap$, $\cup$ and $\setminus$) and find the answers.

(i) the set of men who have had an affair with everyone.

(ii) the set of men who have had an affair with Nora but not with Ophelia.

(iii) the set of men who have had an affair with Penelope but no-one else.

(d) Convert each of the following statements to english and list the members of each set.

(i) $(N \cup P) \cap O$  
(ii) $N \setminus (P \cap O)$  
(iii) $N \cup O \cup P$

(e) Two of the men are hideous mathematicians. Which of the men do you think they are?