Solving Equations

Suppose that we have two or more equations involving a variable $x$, say, and that we want to find a value for $x$. We must isolate $x$ alone on one side of the equation. Most importantly, whatever operation we perform on one side of an equation, we must perform on the other side in order to maintain equality. Remember to check your answer using substitution.

Example

Solve for $x$ given $\frac{2x+6}{3x-2} = 2$.

Get rid of fraction by multiplying both sides by $(3x-2)$:

$$\frac{2x+6}{3x-2} \times (3x-2) = 2 \times (3x-2) \rightarrow 2x+6 = (2 \times 3x) + (2 \times -2) \rightarrow 2x+6 = 6x - 4$$

Subtract $2x$ from each side:

$$2x+6 - 2x = 6x - 4 - 2x \rightarrow 6 = 4x - 4$$

Add 4 to each side:

$$6 + 4 = 4x - 4 + 4 \rightarrow 10 = 4x$$

Divide each side by 4 to get our solution:

$$\frac{10}{4} = \frac{4x}{4} \rightarrow x = \frac{2\frac{1}{2}}{2}$$

The following is a tricky but good question:

Solve for $x$ if $\frac{3}{x-7} = \frac{-4}{-4x+5}$

Invert both fractions: $\frac{x-7}{3} = \frac{-4x+5}{-4}$

Multiply both sides by 3: $\frac{x-7}{3} \times 3 = \frac{-4x+5}{-4} \times 3 \rightarrow x-7 = \frac{3(-4x+5)}{-4}$

Multiply both sides by -4: $(x-7) \times -4 = \frac{3(-4x+5)}{-4} \times -4 \rightarrow -4(x-7) = 3(-4x+5)$

Expand both sides: $-4(x-7) = 3(-4x+5) \rightarrow -4x + 28 = -12x + 15$

Add 4x to both sides: $-4x + 28 + 4x = -12x + 15 + 4x \rightarrow 28 = -8x + 15$

Subtract 15 from both sides: $28 - 15 = -8x + 15 - 15 \rightarrow 13 = -8x$
Divide both sides by -8: \( \frac{13}{-8} = \frac{-8x}{-8} \rightarrow x = \frac{13}{8} \)

Check your answer by substituting \( x = -13/8 \) into \( \frac{3}{x-7} = \frac{-4}{-4x+5} \)!

**Another example:** \(-2x = 6\). To get \( x \) by itself, we must divide both sides by -2. There is a little multiplication sign between the -2 and \( x \), and the opposite of multiplication is division. Then we get \( x = -3 \).

**Another example:** \( \frac{3x}{2} + 4 = 10 \)
The easiest way to do this question is to take 4 from both sides first, then multiply both sides by 2.

\[
\begin{align*}
3x/2 + 4 - 4 &= 10 - 4 \\
3x/2 &= 6 \\
3x/2 \text{ times } 2 &= 6 \text{ times 2} \\
3x &= 12 \\
x &= 4
\end{align*}
\]

**Another example:** \( \frac{x}{2} = x + 4 \)
Multiply both sides by 2
\( x/2 \text{ times } 2 = 2 \text{ times } (x+4) \)
\( x = 2x + 8 \) - we must multiply **both** the \( x \) and the 4 by 2.
\( x - 2x = 2x - 2x + 8 \)
\(-x = 8 \)
\( x = -8 \)

**Solving equations with absolute values**

\[ | -2x - 4 | = 6 \]

The common mistake here is to drop the minus sign in front of the \( 2x \) and 4. DON’T! The only part that changes is the RHS. The LHS stays the same but the RHS has two answers, positive and negative.

Removing the absolute value lines, you get two equations:

\[ -2x - 4 = 6 \quad \text{and} \quad -2x - 4 = -6 \]

Solve these as you would any normal equation.
Multiplying fractions

You DON’T need to find a common denominator. Just multiply the tops together and the bottoms together, then simplify if possible.

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$</th>
<th>$\frac{2}{3} \times \frac{25}{33} = \frac{2 \times 25}{3 \times 33} = \frac{50}{99}$</th>
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Operations on negative numbers

3 - -4 -- here the two negative signs are together, so -- becomes +. Hence the answer is 3 + 4 which is 7.

-3 -4 - here the signs are separated, so we can’t do anything with them. Start at -3 and subtract 4. The answer is -7

Surds

$2 \times \sqrt{7} = 2\sqrt{7}$

$\sqrt{6} \times \sqrt{7} = \sqrt{6 \times 7} = \sqrt{42}$

$\sqrt{4} \times 5 = 5\sqrt{4}$

$\sqrt{6} \times \sqrt{8} = \sqrt{6 \times 8} = \frac{\sqrt{3 \times 2 \times 2 \times 2 \times 2 \times 3}}{6}$

Powers

Anything to the power of 0 equals 1.

$x^0 = 1$  $-2^0 = 1$  $15^0 = 1$

Sigma notation

\[
\sum_{i=1}^{3} i \cdot x = 4
\]

\[
\sum_{i=0}^{2} 2x = 12
\]

\[
\text{When } i = 0, \text{ we just write down } 2x
\]

\[
\text{When } i = 1, \text{ we write down } 2x
\]

\[
\text{When } i = 2, \text{ we write down } 2x
\]

Do we have $2x + 2x + 2x = 12$

\[
6x = 12
\]

\[
x = 2
\]