1. (**1**) First we number the equations for convenience:

$$-5y = -110 + 9x \qquad (1)$$
$$0 = -9y - 63 + 36x \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$9y = 36x - 63 \qquad (3)$$

Dividing both sides of (3) by 9, gives

$$y = 4x - 7 \qquad (4)$$

Substituting for y in equation (1),

$$-5 \times (4x - 7) = -110 + 9x \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} -20x + 35 &= -110 + 9x \\ -29x &= -145 \\ x &= 5 \end{aligned}$$

Next we substitute the value for x into equation (4) to obtain the value for y, giving

$$y = 4 \times 5 - 7 = 13$$

Hence the simultaneous solution to equations (1) and (2) is $(5, 13)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1) \quad \begin{aligned} -5 \times 13 &= -110 + 9 \times 5 \\ -65 &= -110 + 45 \\ -65 &= -65 \end{aligned} \qquad (2) \quad \begin{aligned} 0 &= -9 \times 13 - 63 + 36 \times 5 \\ 0 &= -117 - 63 + 180 \\ 0 &= 0 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) Let $z = \cos x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$-3y - 9z = -9 \qquad (1)$$
$$7y - 8z = -8 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 7 and equation (2) by 3 , giving

$$-21y - 63z = -63 \qquad (3)$$
$$21y - 24z = -24 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-21y + 21y - 63z - 24z = -63 - 24 \qquad (5)$$

Simplifying equation (5) gives

$$-87z = -87 \qquad (6)$$
$$z = 1 \qquad (7)$$

Next we substitute the value for z into equation (1) to obtain the value for y, giving

$$\begin{aligned} -3y - 9 \times 1 &= -9 \\ -3y &= 0 \qquad \text{so} \\ y &= 0 \end{aligned}$$

Now we can find the value of x: $\cos x = 1$, so $x = 0$
Hence the simultaneous solution to equations (1) and (2) is $x = 0$; $y = 0$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\quad -3 \times 0 - 9 \times \cos 0 &= -9 \\ -3 \times 0 - 9 \times 1 &= -9 \\ -9 &= -9 \end{aligned} \qquad \begin{aligned} (2)\quad 7 \times 0 - 8 \times \cos 0 &= -8 \\ 7 \times 0 - 8 \times 1 &= -8 \\ -8 &= -8 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} -7y &= -113 + 3x \qquad (1) \\ -8 - 2x &= -12y \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (2) with x on the left-hand side gives

$$-2x = -12y + 8 \qquad (3)$$

Dividing both sides of (3) by -2, gives

$$x = 6y - 4 \qquad (4)$$

Substituting for x in equation (1),

$$-7y = -113 + 3 \times (6y - 4) \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$\begin{aligned} -7y &= -113 + 18y - 12 \\ -25y &= -125 \\ y &= 5 \end{aligned}$$

Next we substitute the value for y into equation (4) to obtain the value for x, giving

$$x = 6 \times 5 - 4 = 26$$

Hence the simultaneous solution to equations (1) and (2) is $(26, 5)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\quad & -7 \times 5 = -113 + 3 \times 26 \\ & -35 = -113 + 78 \\ & -35 = -35 \end{aligned} \qquad \begin{aligned} (2)\quad & -8 - 2 \times 26 = -12 \times 5 \\ & -8 - 52 = -60 \\ & -60 = -60 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

2. (**1**) First we number the equations for convenience:

$$-7y = -49x - 14 \qquad (1)$$
$$93 = 10y + 3x \qquad (2)$$

We solve these using substitution. Dividing both sides of equation (1) by -7 gives

$$y = 7x + 2 \qquad (3)$$

Substituting for y in equation (2),

$$93 = 10 \times (7x + 2) + 3x \qquad (4)$$

Now (4) is an equation only involving x which gives:

$$93 = 70x + 20 + 3x$$
$$73 = 73x$$
$$1 = x$$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = 7 \times 1 + 2 = 9$$

Hence the simultaneous solution to equations (1) and (2) is $(1, 9)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $-7 \times 9 = -49 \times 1 - 14$
$-63 = -49 - 14$
$-63 = -63$

(2) $93 = 10 \times 9 + 3 \times 1$
$93 = 90 + 3$
$93 = 93$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) Let $z = \sqrt{y}$. Now we have two linear simultaneous equations, which we also number for convenience:

$$-4x - 3z = -24 \qquad (1)$$
$$-13x + 9z = -3 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 , giving

$$-12x - 9z = -72 \qquad (3)$$
$$-13x + 9z = -3 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-12x - 13x - 9z + 9z = -72 - 3 \qquad (5)$$

Simplifying equation (5) gives

$$-25x = -75 \qquad (6)$$
$$x = 3 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for z, giving

$$-4 \times 3 - 3z = -24$$
$$-3z = -12 \qquad \text{so}$$
$$z = 4$$

Now we can find the value of y: $\sqrt{y} = 4$, so $y = 16$
Hence the simultaneous solution to equations (1) and (2) is $x = 3$; $y = 16$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1)\quad \begin{aligned} -4 \times 3 - 3 \times \sqrt{16} &= -24 \\ -4 \times 3 - 3 \times 4 &= -24 \\ -12 - 12 &= -24 \\ -24 &= -24 \end{aligned} \qquad (2)\quad \begin{aligned} -13 \times 3 + 9 \times \sqrt{16} &= -3 \\ -13 \times 3 + 9 \times 4 &= -3 \\ -39 + 36 &= -3 \\ -3 &= -3 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 2y - 2 &= -8x \qquad (1) \\ 0 &= -9y + 18 - 36x \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$9y = -36x + 18 \qquad (3)$$

Dividing both sides of (3) by 9, gives

$$y = -4x + 2 \qquad (4)$$

Substituting for y in equation (1),

$$2 \times (-4x + 2) - 2 = -8x \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} -8x + 4 - 2 &= -8x \\ 2 &= 0 \end{aligned}$$

This statement is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

3. (**1**) First we number the equations for convenience:

$$\begin{aligned} 10y - 488 &= 9x \qquad (1) \\ -2y &= -14x \qquad (2) \end{aligned}$$

We solve these using substitution. Dividing both sides of equation (2) by -2 gives

$$y = 7x \qquad (3)$$

Substituting for y in equation (1),

$$10 \times 7x - 488 = 9x \qquad (4)$$

Now (4) is an equation only involving x which gives:

$$\begin{aligned} 70x - 488 &= 9x \\ 61x &= 488 \\ x &= 8 \end{aligned}$$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = 7 \times 8 = 56$$

Hence the simultaneous solution to equations (1) and (2) is $(8, 56)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $10 \times 56 - 488 = 9 \times 8$
$560 - 488 = 72$
$72 = 72$

(2) $-2 \times 56 = -14 \times 8$
$-112 = -112$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) Let $z = \tan x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$-8y + 3z = 75 \qquad (1)$$
$$-3y + 4z = 31 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by -4 and equation (2) by 3 , giving

$$32y - 12z = -300 \qquad (3)$$
$$-9y + 12z = 93 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-9y + 32y + 12z - 12z = 93 - 300 \qquad (5)$$

Simplifying equation (5) gives

$$23y = -207 \qquad (6)$$
$$y = -9 \qquad (7)$$

Next we substitute the value for y into equation (1) to obtain the value for z, giving

$$-8 \times (-9) + 3z = 75$$
$$3z = 3 \qquad \text{so}$$
$$z = 1$$

Now we can find the value of x: $\tan x = 1$, so $x = \frac{\pi}{4}; \frac{5\pi}{4}$

Hence the simultaneous solution to equations (1) and (2) is $x = \frac{\pi}{4}; \frac{5\pi}{4}; \quad y = -9$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $-8 \times (-9) + 3 \times \tan\frac{\pi}{4} = 75$
$-8 \times (-9) + 3 \times 1 = 75$
$72 + 3 = 75$
$75 = 75$

(2) $-3 \times (-9) + 4 \times \tan\frac{\pi}{4} = 31$
$-3 \times (-9) + 4 \times 1 = 31$
$27 + 4 = 31$
$31 = 31$

We have checked one value of x, you do the other!)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$-10y = 10x + 90 \qquad (1)$$
$$4y + 4x + 36 = 0 \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$4y = -4x - 36 \qquad (3)$$

Dividing both sides of (3) by 4, gives

$$y = -x - 9 \qquad (4)$$

Substituting for y in equation (1),

$$-10 \times (-x - 9) = 10x + 90 \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} 10x + 90 &= 10x + 90 \\ 90 &= 90 \end{aligned}$$

This statement is **always true**, so there is an infinite number of solutions to our simultaneous equations. The lines are superimposed.

4. (**1**) First we number the equations for convenience:

$$\begin{aligned} 4x + 2y &= 2 \qquad (1) \\ 11x - 9y &= -67 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 9 and equation (2) by 2 , giving

$$\begin{aligned} 36x + 18y &= 18 \qquad (3) \\ 22x - 18y &= -134 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$36x + 22x + 18y - 18y = 18 - 134 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} 58x &= -116 \qquad (6) \\ x &= -2 \qquad (7) \end{aligned}$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$\begin{aligned} 4 \times (-2) + 2y &= 2 \\ 2y &= 10 \qquad \text{so} \\ y &= 5 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(-2, 5)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1)\ \begin{aligned} 4 \times (-2) + 2 \times 5 &= 2 \\ -8 + 10 &= 2 \\ 2 &= 2 \end{aligned} \qquad (2)\ \begin{aligned} 11 \times (-2) - 9 \times 5 &= -67 \\ -22 - 45 &= -67 \\ -67 &= -67 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) Let $z = \ln y$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} 8x + 8z &= 0 \qquad (1) \\ 3x - 5z &= -8 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 and equation (2) by -8 , giving

$$\begin{aligned} 24x + 24z &= 0 \qquad (3) \\ -24x + 40z &= 64 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-24x + 24x + 40z + 24z = 64 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} 64z &= 64 \qquad (6) \\ z &= 1 \qquad (7) \end{aligned}$$

Next we substitute the value for z into equation (1) to obtain the value for x, giving

$$\begin{aligned} 8x + 8 \times 1 &= 0 \\ 8x &= -8 \qquad \text{so} \\ x &= -1 \end{aligned}$$

Now we can find the value of y: $\ln y = 1$, so $y = e$

Hence the simultaneous solution to equations (1) and (2) is $x = -1$; $y = e$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 8 \times (-1) + 8 \times \ln e &= 0 \\ 8 \times (-1) + 8 \times 1 &= 0 \\ -8 + 8 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} (2)\ \ 3 \times (-1) - 5 \times \ln e &= -8 \\ 3 \times (-1) - 5 \times 1 &= -8 \\ -3 - 5 &= -8 \\ -8 &= -8 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 3x + 4y &= 54 \qquad (1) \\ -13x + 7y &= -88 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by -7 and equation (2) by 4 , giving

$$\begin{aligned} -21x - 28y &= -378 \qquad (3) \\ -52x + 28y &= -352 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-21x - 52x - 28y + 28y = -378 - 352 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -73x &= -730 \qquad (6) \\ x &= 10 \qquad (7) \end{aligned}$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$\begin{aligned} 3 \times 10 + 4y &= 54 \\ 4y &= 24 \qquad \text{so} \\ y &= 6 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(10, 6)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1)\ \ 3 \times 10 + 4 \times 6 = 54 \qquad (2)\ \ -13 \times 10 + 7 \times 6 = -88$$
$$30 + 24 = 54 \qquad -130 + 42 = -88$$
$$54 = 54 \qquad -88 = -88$$

Both equations turned into true statements, as required. Hence the answer is correct.)

5. (**1**) First we number the equations for convenience:

$$-3x - 2y = -14 \qquad (1)$$
$$30x + 20y = 154 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 10 , giving

$$-30x - 20y = -140 \qquad (3)$$
$$30x + 20y = 154 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-30x + 30x - 20y + 20y = -140 + 154 \qquad (5)$$

Simplifying equation (5) gives

$$0 = 14 \qquad (6)$$

Statement (6) is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

(**2**) Let $z = \ln y$. Now we have two linear simultaneous equations, which we also number for convenience:

$$2z - 4x = 16 \qquad (1)$$
$$10z + 9x = -36 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by -5 , giving

$$-10z + 20x = -80 \qquad (3)$$
$$10z + 9x = -36 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-10z + 10z + 20x + 9x = -80 - 36 \qquad (5)$$

Simplifying equation (5) gives

$$29x = -116 \qquad (6)$$
$$x = -4 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for z, giving

$$2z - 4 \times (-4) = 16$$
$$2z = 0 \qquad \text{so}$$
$$z = 0$$

Now we can find the value of y: $\ln y = 0$, so $y = 1$

Hence the simultaneous solution to equations (1) and (2) is $x = -4; \quad y = 1$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\;\; 2 \times \ln 1 - 4 \times (-4) &= 16 \\ 2 \times 0 - 4 \times (-4) &= 16 \\ 16 &= 16 \end{aligned} \qquad\qquad \begin{aligned} (2)\;\; 10 \times \ln 1 + 9 \times (-4) &= -36 \\ 10 \times 0 + 9 \times (-4) &= -36 \\ -36 &= -36 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 0 &= -63 - 9x + 81y \qquad (1) \\ 31 - 45y &= -5x \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (1) with x on the left-hand side gives

$$9x = 81y - 63 \qquad (3)$$

Dividing both sides of (3) by 9, gives

$$x = 9y - 7 \qquad (4)$$

Substituting for x in equation (2),

$$31 - 45y = -5 \times (9y - 7) \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$\begin{aligned} 31 - 45y &= -45y + 35 \\ 31 &= 35 \end{aligned}$$

This statement is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.