

1. (1) Let $u = 9x^8 + 2$, so $y = u^2$.

$$\text{Now } \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}.$$

$$\frac{dy}{du} = 2 \times u^{2-1} = 2u^1 = 2u$$

$$\frac{du}{dx} = 9 \times 8 \times x^{8-1} = 72x^7$$

$$\text{So, } \frac{dy}{dx} = 2u \times 72x^7 = 2(9x^8 + 2) \times 72x^7 = 144x^7(9x^8 + 2).$$

$$\text{Hence } \frac{dy}{dx} = 144x^7(9x^8 + 2).$$

- (2) Let $u = 6 - 9x$, then $u' = -9$.

$$\text{Let } v = 9 - 8x, \text{ then } v' = -8.$$

$$\text{Quotient rule: } y' = \frac{u'v - uv'}{v^2}, \text{ so}$$

$$y' = \frac{-9 \times (9 - 8x) - (6 - 9x) \times (-8)}{(9 - 8x)^2} = \frac{-81 + 72x + 48 - 72x}{(9 - 8x)^2} = -\frac{33}{(9 - 8x)^2}.$$

$$\text{Hence } y' = -\frac{33}{(9 - 8x)^2}.$$

- (3) Let $u = -2z^3 + 7$, then $u' = -6z^2$.

$$\text{Let } v = -9 - 9z, \text{ then } v' = -9.$$

$$\text{Product rule: } y' = u'v + uv'.$$

Substitute u, u', v and v' into the product rule:

$$\begin{aligned} y' &= -6z^2 \times (-9 - 9z) + (-2z^3 + 7) \times (-9) \\ &= 54z^2 + 54z^3 + 18z^3 - 63 \end{aligned}$$

$$\text{Hence } y' = 72z^3 + 54z^2 - 63.$$

- (4) Let $u = 4t^2 + 9$, then $u' = 8t$.

$$\text{Let } v = -9t^2 + 5t, \text{ then } v' = -18t + 5.$$

$$\text{Quotient rule: } y' = \frac{u'v - uv'}{v^2}, \text{ so}$$

$$\begin{aligned} y' &= \frac{8t \times (-9t^2 + 5t) - (4t^2 + 9) \times (-18t + 5)}{(-9t^2 + 5t)^2} \\ &= \frac{-72t^3 + 40t^2 - (-72t^3 + 20t^2 - 162t + 45)}{(-9t^2 + 5t)^2} \\ &= \frac{-72t^3 + 40t^2 + 72t^3 - 20t^2 + 162t - 45}{(-9t^2 + 5t)^2} \end{aligned}$$

$$\text{Hence } y' = \frac{20t^2 + 162t - 45}{(-9t^2 + 5t)^2}.$$

- (5) Let $u = -5z - 4z^2 + 2$, then $u' = -5 - 8z$.

$$\text{Let } v = -1 - 5z, \text{ then } v' = -5.$$

$$\text{Quotient rule: } y' = \frac{u'v - uv'}{v^2}, \text{ so}$$

$$y' = \frac{(-5-8z) \times (-1-5z) - (-5z-4z^2+2) \times (-5)}{(-1-5z)^2}$$

$$= \frac{5+25z+8z+40z^2-25z-20z^2+10}{(-1-5z)^2}$$

Hence $y' = \frac{20z^2+8z+15}{(-1-5z)^2}$.

2. (1) Let $u = 3x^{-8} + 6$, so $y = u^2$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$$\frac{dy}{du} = 2 \times u^{2-1} = 2u^1 = 2u$$

$$\frac{du}{dx} = 3 \times (-8) \times x^{-8-1} = -24x^{-9}$$

So, $\frac{dy}{dx} = 2u \times (-24x^{-9}) = 2(3x^{-8} + 6) \times (-24x^{-9}) = -48x^{-9}(3x^{-8} + 6) = -\frac{48(3x^{-8} + 6)}{x^9}$.

Hence $\frac{dy}{dx} = -\frac{48(3x^{-8} + 6)}{x^9}$.

- (2) Let $u = 6z - 6$, then $u' = 6$.

Let $v = -9z - 6$, then $v' = -9$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{6 \times (-9z - 6) - (6z - 6) \times (-9)}{(-9z - 6)^2} = \frac{-54z - 36 + 54z - 54}{(-9z - 6)^2} = -\frac{90}{(-9z - 6)^2}$$

Hence $y' = -\frac{90}{(-9z - 6)^2}$.

- (3) Let $u = -5 - 6t^3$, then $u' = -18t^2$.

Let $v = -6 + t$, then $v' = 1$.

Product rule: $y' = u'v + uv'$.

Substitute u, u', v and v' into the product rule:

$$y' = -18t^2 \times (-6 + t) + (-5 - 6t^3) \times 1$$

$$= 108t^2 - 18t^3 - 5 - 6t^3$$

Hence $y' = -24t^3 + 108t^2 - 5$.

- (4) Let $u = -10z^2 + z$, then $u' = -20z + 1$.

Let $v = 7z^2 + 6z$, then $v' = 14z + 6$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{(-20z+1) \times (7z^2+6z) - (-10z^2+z) \times (14z+6)}{(7z^2+6z)^2}$$

$$= \frac{-140z^3 - 120z^2 + 7z^2 + 6z - (-140z^3 - 60z^2 + 14z^2 + 6z)}{(7z^2+6z)^2}$$

$$= \frac{-140z^3 - 120z^2 + 7z^2 + 6z + 140z^3 + 60z^2 - 14z^2 - 6z}{(7z^2+6z)^2}$$

Hence $y' = \frac{-67z^2}{(7z^2+6z)^2}$.

- (5) Let $u = -4z^2 - 9z - 3$, then $u' = -8z - 9$.
 Let $v = -7z^2 + 5z + 3$, then $v' = -14z + 5$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{(-8z - 9) \times (-7z^2 + 5z + 3) - (-4z^2 - 9z - 3) \times (-14z + 5)}{(-7z^2 + 5z + 3)^2}$$

$$= \frac{56z^3 - 40z^2 - 24z + 63z^2 - 45z - 27 - 56z^3 + 20z^2 - 126z^2 + 45z - 42z + 15}{(-7z^2 + 5z + 3)^2}$$

Hence $y' = \frac{-83z^2 - 66z - 12}{(-7z^2 + 5z + 3)^2}$.

3. (1) Let $u = -7 - 9x^4$, so $y = u^2$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$$\frac{dy}{du} = 2 \times u^{2-1} = 2u^1 = 2u$$

$$\frac{du}{dx} = -9 \times 4 \times x^{4-1} = -36x^3$$

So, $\frac{dy}{dx} = 2u \times (-36x^3) = 2(-7 - 9x^4) \times (-36x^3) = -72x^3(-7 - 9x^4)$.

Hence $\frac{dy}{dx} = -72x^3(-7 - 9x^4)$.

- (2) Let $u = -2x$, then $u' = -2$.

Let $v = -5x + 1$, then $v' = -5$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{-2 \times (-5x + 1) - (-2x \times (-5))}{(-5x + 1)^2} = \frac{10x - 2 - 10x}{(-5x + 1)^2} = -\frac{2}{(-5x + 1)^2}$$

Hence $y' = -\frac{2}{(-5x + 1)^2}$.

- (3) Let $u = -5 - 3h$, then $u' = -3$.

Let $v = -h + h^3$, then $v' = -1 + 3h^2$.

Product rule: $y' = u'v + uv'$.

Substitute u, u', v and v' into the product rule:

$$y' = -3 \times (-h + h^3) + (-5 - 3h) \times (-1 + 3h^2)$$

$$= 3h - 3h^3 + 5 - 15h^2 + 3h - 9h^3$$

Hence $y' = -12h^3 - 15h^2 + 6h + 5$.

- (4) Let $u = 7x^2 + 5x$, then $u' = 14x + 5$.

Let $v = 7x^2 + 2$, then $v' = 14x$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{(14x + 5) \times (7x^2 + 2) - (7x^2 + 5x) \times 14x}{(7x^2 + 2)^2}$$

$$= \frac{98x^3 + 28x + 35x^2 + 10 - (98x^3 + 70x^2)}{(7x^2 + 2)^2}$$

$$= \frac{98x^3 + 28x + 35x^2 + 10 - 98x^3 - 70x^2}{(7x^2 + 2)^2}$$

Hence $y' = \frac{-35x^2 + 28x + 10}{(7x^2 + 2)^2}$.

(5) Let $u = -1 + 8r - 7r^2$, then $u' = 8 - 14r$.

Let $v = -9r^2 - 6r$, then $v' = -18r - 6$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{(8 - 14r) \times (-9r^2 - 6r) - (-1 + 8r - 7r^2) \times (-18r - 6)}{(-9r^2 - 6r)^2} \\ &= \frac{-72r^2 - 48r + 126r^3 + 84r^2 - 18r - 6 + 144r^2 + 48r - 126r^3 - 42r^2}{(-9r^2 - 6r)^2} \end{aligned}$$

Hence $y' = \frac{114r^2 - 18r - 6}{(-9r^2 - 6r)^2}$.

4. (1) Let $u = 9 - x^{-3}$, so $y = \frac{1}{u^3} = u^{-3}$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$$\frac{dy}{du} = -3 \times u^{-3-1} = -3u^{-4}$$

$$\frac{du}{dx} = 3 \times x^{-3-1} = 3x^{-4}$$

So, $\frac{dy}{dx} = -3u^{-4} \times 3x^{-4} = -3(9 - x^{-3})^{-4} \times 3x^{-4} = -9x^{-4}(9 - x^{-3})^{-4} = -\frac{9}{x^4(9 - x^{-3})^4}$.

Hence $\frac{dy}{dx} = -\frac{9}{x^4(9 - x^{-3})^4}$.

(2) Let $u = 3t + 5$, then $u' = 3$.

Let $v = 8 - t$, then $v' = -1$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{3 \times (8 - t) - (3t + 5) \times (-1)}{(8 - t)^2} = \frac{24 - 3t + 3t + 5}{(8 - t)^2} = \frac{29}{(8 - t)^2}$$

Hence $y' = \frac{29}{(8 - t)^2}$.

(3) Let $u = 4h^2 - 1$, then $u' = 8h$.

Let $v = 4h^3 - 2h^2$, then $v' = 12h^2 - 4h$.

Product rule: $y' = u'v + uv'$.

Substitute u, u', v and v' into the product rule:

$$\begin{aligned} y' &= 8h \times (4h^3 - 2h^2) + (4h^2 - 1) \times (12h^2 - 4h) \\ &= 32h^4 - 16h^3 + 48h^4 - 16h^3 - 12h^2 + 4h \end{aligned}$$

Hence $y' = 80h^4 - 32h^3 - 12h^2 + 4h$.

(4) Let $u = 7h^2 + 3$, then $u' = 14h$.

Let $v = 4h^2 + 5$, then $v' = 8h$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$\begin{aligned}
y' &= \frac{14h \times (4h^2 + 5) - (7h^2 + 3) \times 8h}{(4h^2 + 5)^2} \\
&= \frac{56h^3 + 70h - (56h^3 + 24h)}{(4h^2 + 5)^2} \\
&= \frac{56h^3 + 70h - 56h^3 - 24h}{(4h^2 + 5)^2}
\end{aligned}$$

Hence $y' = \frac{46h}{(4h^2 + 5)^2}$.

- (5) Let $u = -3 + 2r$, then $u' = 2$.
Let $v = r + 6r^2 - 4$, then $v' = 1 + 12r$.
Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$\begin{aligned}
y' &= \frac{2 \times (r + 6r^2 - 4) - (-3 + 2r) \times (1 + 12r)}{(r + 6r^2 - 4)^2} \\
&= \frac{2r + 12r^2 - 8 + 3 + 36r - 2r - 24r^2}{(r + 6r^2 - 4)^2}
\end{aligned}$$

Hence $y' = \frac{-12r^2 + 36r - 5}{(r + 6r^2 - 4)^2}$.

5. (1) Let $u = -4x^6 - 3$, so $y = u^5$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$$\frac{dy}{du} = 5 \times u^{5-1} = 5u^4$$

$$\frac{du}{dx} = -4 \times 6 \times x^{6-1} = -24x^5$$

So, $\frac{dy}{dx} = 5u^4 \times (-24x^5) = 5(-4x^6 - 3)^4 \times (-24x^5) = -120x^5(-4x^6 - 3)^4$.

Hence $\frac{dy}{dx} = -120x^5(-4x^6 - 3)^4$.

- (2) Let $u = 3 - 2r$, then $u' = -2$.
Let $v = 7 + 9r$, then $v' = 9$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$y' = \frac{-2 \times (7 + 9r) - (3 - 2r) \times 9}{(7 + 9r)^2} = \frac{-14 - 18r - 27 + 18r}{(7 + 9r)^2} = -\frac{41}{(7 + 9r)^2}$$

Hence $y' = -\frac{41}{(7 + 9r)^2}$.

- (3) Let $u = -2r + 10r^3$, then $u' = -2 + 30r^2$.
Let $v = -3r^2 + 4$, then $v' = -6r$.

Product rule: $y' = u'v + uv'$.

Substitute u, u', v and v' into the product rule:

$$\begin{aligned}
y' &= (-2 + 30r^2) \times (-3r^2 + 4) + (-2r + 10r^3) \times (-6r) \\
&= 6r^2 - 8 - 90r^4 + 120r^2 + 12r^2 - 60r^4
\end{aligned}$$

Hence $y' = -150r^4 + 138r^2 - 8$.

(4) Let $u = -6t^2 + 5t$, then $u' = -12t + 5$.

Let $v = -t^2 + 4$, then $v' = -2t$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{(-12t + 5) \times (-t^2 + 4) - (-6t^2 + 5t) \times (-2t)}{(-t^2 + 4)^2} \\ &= \frac{12t^3 - 48t - 5t^2 + 20 - (12t^3 - 10t^2)}{(-t^2 + 4)^2} \\ &= \frac{12t^3 - 48t - 5t^2 + 20 - 12t^3 + 10t^2}{(-t^2 + 4)^2} \end{aligned}$$

$$\text{Hence } y' = \frac{5t^2 - 48t + 20}{(-t^2 + 4)^2}.$$

(5) Let $u = 7t + 7t^2 + 1$, then $u' = 7 + 14t$.

Let $v = -8t + 6t^2 - 3$, then $v' = -8 + 12t$.

Quotient rule: $y' = \frac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{(7 + 14t) \times (-8t + 6t^2 - 3) - (7t + 7t^2 + 1) \times (-8 + 12t)}{(-8t + 6t^2 - 3)^2} \\ &= \frac{-56t + 42t^2 - 21 - 112t^2 + 84t^3 - 42t + 56t - 84t^2 + 56t^2 - 84t^3 + 8 - 12t}{(-8t + 6t^2 - 3)^2} \end{aligned}$$

$$\text{Hence } y' = \frac{-98t^2 - 54t - 13}{(-8t + 6t^2 - 3)^2}.$$