MATH1040/7040 Week 8 SOLUTIONS

1. To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

(**1**) $6x(-6+7x) = 0$, so

$$\begin{aligned} 6x &= 0 \qquad \textit{or} \qquad & -6+7x &= 0 \\ x &= 0 & 7x &= 6 \\ & & x &= \frac{6}{7} \end{aligned}$$

(**2**) $(-2+y)(-8y+3) = 0$, so

$$\begin{aligned} -2+y &= 0 \qquad \textit{or} \qquad & -8y+3 &= 0 \\ y &= 2 & -8y &= -3 \\ & & y &= \frac{3}{8} \end{aligned}$$

(**3**) $2(-3z+3)(8z-7) = 0$, so

$$\begin{aligned} -3z+3 &= 0 \qquad \textit{or} \qquad & 8z-7 &= 0 \\ -3z &= -3 & 8z &= 7 \\ z &= \frac{-3}{-3} & z &= \frac{7}{8} \\ z &= 1 \end{aligned}$$

(**4**) $(-8-7x)^9 = 0$, so $-8-7x = 0$, so $-7x = 8$, so $x = -\frac{8}{7}$

2. $6y(-5y-6) = 0$, so

$$\begin{aligned} 6y &= 0 \qquad \textit{or} \qquad & -5y-6 &= 0 \\ y &= 0 & -5y &= 6 \\ & & y &= -\frac{6}{5} \end{aligned}$$

3. $f(x) = x^2 + 3x - 1$, so
$f(-8) = (-8)^2 + 3 \times (-8) - 1 = 64 - 24 - 1 = 39$

4. $5z^2 - 35z + 60 = 0$, so we use $a = 5, b = -35, c = 60$ in the quadratic formula. Hence

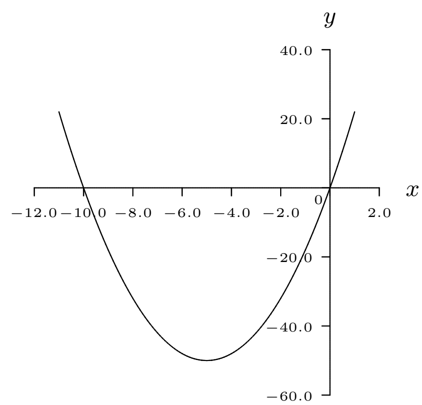
$$\begin{aligned} z &= \frac{35 \pm \sqrt{(-35)^2 - 4 \times 5 \times 60}}{2 \times 5} \\ &= \frac{35 \pm \sqrt{1225 - 1200}}{10} \\ &= \frac{35 \pm \sqrt{25}}{10} \\ &= \frac{35+5}{10} \; \textit{or} \; \frac{35-5}{10} \\ &= \frac{40}{10} \; \textit{or} \; \frac{30}{10} \\ &= 4 \; \textit{or} \; 3 \end{aligned}$$

5. (a) The roots of $y = 2x^2 + 20x$ are the x values that satisfy $2x^2 + 20x = 0$. You can solve this equation either by using the quadratic formula or by factoring. Here we will use factoring.

First divide through by 2 to get $x^2 + 10x = 0$. Now because $x^2 + 10x = (x + 10)\,x$, the two roots of the quadratic equation are $x = -10,\ 0$.

(b) The y-intercept occurs when $x = 0$, so substituting this into $y = 2x^2 + 20x$ gives $y = 0$.

(c)



6. Let P be the amount invested, r be the interest rate per time period, n be the number of time periods and F be the final value. In each case, $P = 400$. Then:

(1) Interest compounds annually, so we use the rate and number of time periods given in the question.
Hence $r = 6.0\% = 0.06$ and $n = 6$, so $F = 400 \times (1 + 0.06)^6 = 400 \times 1.06^6 \approx 567.41$.
The final balance is \$567.41.

(2) Interest compounds twice a year, so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 3.0\% = 0.03$ and $n = 12$, so $F = 400 \times (1 + 0.03)^{12} = 400 \times 1.03^{12} \approx 570.30$.
The final balance is \$570.30.

(3) Interest compounds 4 times a year, so we need to divide the given rate by 4 and multiply the given number of years by 4.
Hence $r = 1.5\% = 0.015$ and $n = 24$, so $F = 400 \times (1 + 0.015)^{24} = 400 \times 1.015^{24} \approx 571.80$.
The final balance is \$571.80.

(4) Interest compounds 12 times a year, so we need to divide the given rate by 12 and multiply the given number of years by 12.
Hence $r = 0.5\% = 0.005$ and $n = 72$, so $F = 400 \times (1 + 0.005)^{72} = 400 \times 1.005^{72} \approx 572.82$.
The final balance is \$572.82.

(5) Interest compounds continuously, so $F = 400e^{0.06\times 6} = 400e^{0.36} \approx 573.33$.
The final balance is \$573.33.

7. **(1)** $y = -4 \times |-4x|$, so $y = -4 \times |4x|$, which is a graph of negative absolute value. Hence the matching graph is Graph M.

(2) $-12y = -13y + 1$, so $y = 1$. Hence this is a horizontal line, with y positive. Hence the matching graph is Graph C.

(3) $7y + 3x - 15 = -7x - 16$, so $7y = -10x - 1$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

(**4**) $y = e^{7x}$, which is a graph of exponential growth. Hence the matching graph is Graph K.

(**5**) $-11y + 3 = 15x^2 - 8$, so $11y = -15x^2 + 11$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is positive. Hence the matching graph is Graph R.

(**6**) $2y + 4x^2 - 13 = 8y + 7x^2 - 13$, so $6y = -3x^2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is 0. Hence the matching graph is Graph S.

(**7**) $3y - 2x^2 - 12 = 2y + 6$, so $y = 2x^2 + 18$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is positive. Hence the matching graph is Graph O.

(**8**) $-5y = -15y - 5x$, so $10y = -5x$. Hence this is a straight line, with negative gradient and passing through the origin. Hence the matching graph is Graph I.