MATH1040/7040 Week 7 SOLUTIONS

1. Let $(x_1, y_1) = (-1, \sqrt{2})$ and $(x_2, y_2) = (10, \sqrt{2})$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-1-10)^2 + \left(\sqrt{2} - \sqrt{2}\right)^2} = \sqrt{(-11)^2 + 0^2} = \sqrt{121 + 0} = \sqrt{121}$.
Hence $d = 11$

2. First we number the equations for convenience:

$$-8y + 7x = 38 \qquad (1)$$
$$3y - 8x = -25 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 and equation (2) by 8 , giving

$$-24y + 21x = 114 \qquad (3)$$
$$24y - 64x = -200 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$24y - 24y - 64x + 21x = -200 + 114 \qquad (5)$$

Simplifying equation (5) gives

$$-43x = -86 \qquad (6)$$
$$x = 2 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$-8y + 7 \times 2 = 38$$
$$-8y = 24 \qquad \text{so}$$
$$y = -3$$

Hence the simultaneous solution to equations (1) and (2) is $(2, -3)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1) \quad -8 \times (-3) + 7 \times 2 = 38$$
$$24 + 14 = 38$$
$$38 = 38$$

$$(2) \quad 3 \times (-3) - 8 \times 2 = -25$$
$$-9 - 16 = -25$$
$$-25 = -25$$

Both equations turned into true statements, as required. Hence the answer is correct.)

3. First we number the equations for convenience:

$$-30 = -6x - 30y \qquad (1)$$
$$14 + 4x = -3y \qquad (2)$$

We solve these using substitution. Rearranging equation (1) with x on the right-hand side gives

$$30y - 30 = -6x \qquad (3)$$

Dividing both sides of (3) by -6 gives

$$-5y + 5 = x \qquad (4)$$

Substituting for x in equation (2),

$$14 + 4 \times (-5y + 5) = -3y \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$\begin{aligned} 14 - 20y + 20 &= -3y \\ -17y &= -34 \\ y &= 2 \end{aligned}$$

Next we substitute the value for y into equation (4) to obtain the value for x, giving

$$x = -5 \times 2 + 5 = -5$$

Hence the simultaneous solution to equations (1) and (2) is $(-5, 2)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad & -30 = -6 \times (-5) - 30 \times 2 \\ & -30 = 30 - 60 \\ & -30 = -30 \end{aligned} \qquad \begin{aligned} (2) \quad & 14 + 4 \times (-5) = -3 \times 2 \\ & 14 - 20 = -6 \\ & -6 = -6 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

4. We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} -3y - 3x &= 21 \qquad (1) \\ -8y - 4x &= 28 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by -4 and equation (2) by 3 , giving

$$\begin{aligned} 12y + 12x &= -84 \qquad (3) \\ -24y - 12x &= 84 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$12y - 24y + 12x - 12x = -84 + 84 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -12y &= 0 \qquad (6) \\ y &= 0 \qquad (7) \end{aligned}$$

Next we substitute the value for y into equation (1) to obtain the value for x, giving

$$\begin{aligned} -3 \times 0 - 3x &= 21 \\ -3x &= 21 \qquad \text{so} \\ x &= -7 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(-7, 0)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad -3 \times 0 - 3 \times (-7) &= 21 \\ 21 &= 21 \end{aligned} \qquad \begin{aligned} (2) \quad -8 \times 0 - 4 \times (-7) &= 28 \\ 28 &= 28 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

5. We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} -210 + 3x &= -6y \qquad (1) \\ -90y + 70 &= -10x \qquad (2) \end{aligned}$$

We solve these using substitution. Dividing both sides of equation (2) by -10 gives

$$9y - 7 = x \qquad (3)$$

Substituting for x in equation (1),

$$-210 + 3 \times (9y - 7) = -6y \qquad (4)$$

Now (4) is an equation only involving y which gives:

$$\begin{aligned} -210 + 27y - 21 &= -6y \\ 33y &= 231 \\ y &= 7 \end{aligned}$$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = 9 \times 7 - 7 = 56$$

Hence the simultaneous solution to equations (1) and (2) is $(56, 7)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $-210 + 3 \times 56 = -6 \times 7$
$-210 + 168 = -42$
$-42 = -42$

(2) $-90 \times 7 + 70 = -10 \times 56$
$-630 + 70 = -560$
$-560 = -560$

Both equations turned into true statements, as required. Hence the answer is correct.)

6. $f(z) = \left|\dfrac{6}{3z}\right|$
When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;
- denominator of a fraction cannot be 0, so $3z \neq 0$.

Hence, the domain of this function is $(-\infty, 0) \cup (0, \infty)$, i.e. $z \neq 0$.

7. $f(w) = \dfrac{9}{w^2 - 2}$
When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $w^2 - 2 \neq 0$;
- so $w^2 \neq 2$;
- we can square any number and result will always be a positive number or 0, so $w \neq \pm\sqrt{2}$.

Hence, the domain of this function is $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$, i.e. $w \neq \pm\sqrt{2}$.

8. $f(z) = \left|(\sqrt{z})^2\right|$
When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;

- we can square any number;
- we can only take the square root of positive numbers or 0, so $z \geq 0$.

Hence, the domain of this function is $[0, \infty)$, i.e. $z \geq 0$.

When evaluating the range, we need to keep in mind the following (starting with variable z):

- square root is always positive or 0, so $\sqrt{z} \geq 0$;
- squaring always gives a positive or 0, so $\left(\sqrt{z}\right)^2 \geq 0$;
- absolute value is always positive or 0, so $\left|\left(\sqrt{z}\right)^2\right| \geq 0$.

Hence, the range of this function is $[0, \infty)$.

9. $f(w) = -\left|\sqrt{w}\right|$
When evaluating the range, we need to keep in mind the following (starting with variable w):

- square root is always positive or 0, so $\sqrt{w} \geq 0$;
- absolute value is always positive or 0, so $\left|\sqrt{w}\right| \geq 0$;
- multiplying by a negative number usually reverses the inequality , so $-\left|\sqrt{w}\right| \leq 0$.

Hence, the range of this function is $(-\infty, 0]$.

10. $f(x) = \dfrac{2}{x^2 - 10}$
When evaluating the range, we need to keep in mind the following (starting with variable x):

- squaring always gives a positive or 0, so $0 \leq x^2$;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
- so $-10 \leq x^2 - 10$ and $x^2 - 10 \neq 0$.

Hence, the range of this function is $(-\infty, -\frac{1}{5}] \cup (0, \infty)$.

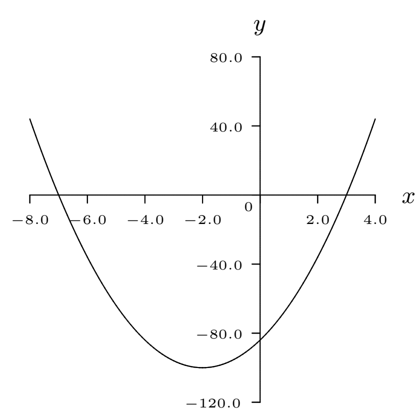
11. $f(z) = -z^2 - 2z + 3$, so
$f(-5) = -(-5)^2 - 2 \times (-5) + 3 = -25 + 10 + 3 = -12$

12. (a) The roots of $y = 4x^2 + 16x - 84$ are the x values that satisfy $4x^2 + 16x - 84 = 0$. You can solve this equation either by using the quadratic formula or by factoring. Here we will use factoring.

First divide through by 4 to get $x^2 + 4x - 21 = 0$. Now because $x^2 + 4x - 21 = (x + 7)(x - 3)$, the two roots of the quadratic equation are $x = -7, 3$.

(b) The y-intercept occurs when $x = 0$, so substituting this into $y = 4x^2 + 16x - 84$ gives $y = -84$.

(c)



13. $4y^2 - 4y - 80 = 0$, so we use $a = 4, b = -4, c = -80$ in the quadratic formula. Hence

$$
\begin{aligned}
y &= \frac{4 \pm \sqrt{(-4)^2 - 4 \times 4 \times (-80)}}{2 \times 4} \\
&= \frac{4 \pm \sqrt{16 - (-1280)}}{8} \\
&= \frac{4 \pm \sqrt{1296}}{8} \\
&= \frac{4 + 36}{8} \quad or \quad \frac{4 - 36}{8} \\
&= \frac{40}{8} \quad or \quad \frac{-32}{8} \\
&= 5 \quad or \quad -4
\end{aligned}
$$

14. To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

(1) $8y\,(10y - 7) = 0$, so

$$
\begin{aligned}
8y &= 0 & \quad or \quad && 10y - 7 &= 0 \\
y &= 0 & && 10y &= 7 \\
& & && y &= \frac{7}{10}
\end{aligned}
$$

(2) $(5x + 3)\,(-10 + x) = 0$, so

$$
\begin{aligned}
5x + 3 &= 0 & \quad or \quad && -10 + x &= 0 \\
5x &= -3 & && x &= 10 \\
x &= -\frac{3}{5} & && &
\end{aligned}
$$

(3) $5\,(-3 + 7z)\,(-1 + 7z) = 0$, so

$$
\begin{aligned}
-3 + 7z &= 0 & \quad or \quad && -1 + 7z &= 0 \\
7z &= 3 & && 7z &= 1 \\
z &= \frac{3}{7} & && z &= \frac{1}{7}
\end{aligned}
$$

(4) $(-4z + 5)^2 = 0$, so $-4z + 5 = 0$, so $-4z = -5$, so $z = \frac{5}{4}$

15. $8\,(7z - 9)\,(3z - 3) = 0$, so

$$
\begin{aligned}
7z - 9 &= 0 & \quad or \quad && 3z - 3 &= 0 \\
7z &= 9 & && 3z &= 3 \\
z &= \frac{9}{7} & && z &= \frac{3}{3} \\
& & && z &= 1
\end{aligned}
$$