## MATH1040/7040 WEEK 5 SOLUTIONS

**1.** $\sum_{i=-2}^{2} 5i^2 = 5\times(-2)^2 + 5\times(-1)^2 + 5\times 0^2 + 5\times 1^2 + 5\times 2^2 = 20+5+0+5+20 = 50$

Hence $z$=50

**2.** $\sum_{j=0}^{6} (-2)^j\, j = (-2)^0\times 0 + (-2)^1\times 1 + (-2)^2\times 2 + (-2)^3\times 3 + (-2)^4\times 4 + (-2)^5\times 5 + (-2)^6\times 6 = 0-2+8-24+64-160+384 = 270$

**3.** $\sum_{i=3}^{4} xi = -7,$ so $3x+4x=-7,$ so $7x=-7$

Hence $x=-1$

**4.** $\sum_{i=1}^{3} -3x = 0,$ so $-3x-3x-3x=0,$ so $-9x=0$

Hence $x=0$

**5.** $\frac{6}{2}+\frac{6}{3}+\frac{6}{4}+\frac{6}{5} = \sum_{i=2}^{5}\frac{6}{i}$

**6.** $x^2+4x^3+9x^4+16x^5+\ldots = \sum_{i=1}^{\infty} i^2x^{i+1}$

**7.** To determine whether the given line passes through the point $(x_1, y_1) = (5, -9)$, we need to substitute the coordinates of the point into the equation of the line. Now,

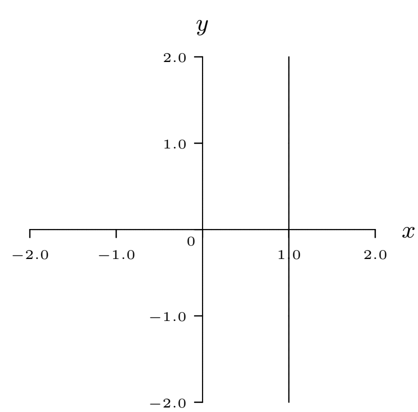
$$\begin{aligned} 9y &= 81+27x, \text{ so} \\ 9\times(-9) &= 81+27\times 5 \\ -81 &= 81+135 \\ -81 &= 216 \end{aligned}$$

The last statement is **not true**, so our line **does not** pass through the point $(5, -9)$.

**8.** (a) First we rearrange the equation to get $x=1$. Therefore, $x=1$ regardless of the value of $y$. Hence, the line does not intercept the $y$-axis at all and there is no $y$-intercept.

(b) The line $x=1$ has constant $x$-value. Hence, the $x$-intercept is $x=1$.

(c)



**9.** Rewrite the equation as $y = mx + c$ :

$$\begin{aligned} 0 &= 9y - 8x, \quad \text{so} \\ -9y &= -8x \\ y &= \frac{8}{9}x \end{aligned}$$

Hence the gradient is $m = \frac{8}{9}$ and the $y$-intercept is $c = 0$.

**10.** Rewrite the equation as $y = mx + c$ :

$$\begin{aligned} 3x - 5 - 2y &= -5y - 3x, \text{ so} \\ -2y + 5y &= -3x - 3x + 5 \\ 3y &= -6x + 5 \\ y &= -2x + \frac{5}{3} \end{aligned}$$

Hence the gradient is $m = -2$ and the $y$-intercept is $c = \frac{5}{3}$.

**11.** Let $(x_1, y_1) = (-6, 0)$ and $(x_2, y_2) = (-5, 10)$. To find the equation of the line through $(x_1, y_1)$ and $(x_2, y_2)$ you must find the gradient $m$ and the $y$-intercept $c$.
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{-5 - (-6)} = \frac{10}{1}$. Hence $m = 10$.
Thus the equation of the line is $y = 10x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-6, 0)$ into this equation to get the value for $c$.
Hence $0 = 10 \times (-6) + c$, so $0 = -60 + c$. Hence $c = 0 - (-60) = 60$.
Hence the equation of the line is $y = 10x + 60$.

**12.** Thus the equation of the line is $y = 1x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (2, 0)$ into this equation to get the value for $c$. Hence $0 = 1 \times 2 + c$ , so $-2 = c$.
Hence the equation of the line is $y = x - 2$.

**13.** To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4x - 7 + y &= -51 + 15x - 10y, \text{ so} \\ y + 10y &= 15x - 4x - 51 + 7 \\ 11y &= 11x - 44 \\ y &= x - 4 \end{aligned}$$

Hence, the gradient of the original line is $m = 1$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-8, -1)$ into this equation to get the value for $c$.
$-1 = 1 \times (-8) + c$, so $-1 = -8 + c$. Hence $c = -1 - (-8) = 7$.
Hence the equation of the line is $y = x + 7$.

**14.** To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4y + 8 &= 12x, \text{ so} \\ 4y &= 12x - 8 \\ y &= 3x - 2 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 3$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -\frac{1}{3}$.

Thus the equation of the line is $y = -\frac{1}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-24, 4)$ into this equation to get the value of $c$:
$4 = -\frac{1}{3} \times (-24) + c$, so $4 = 8 + c$. Hence $c = 4 - 8 = -4$.
Hence the equation of the line is $y = -\frac{1}{3}x - 4$.

**15.** To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 35 &= -7y, \text{ so} \\ 7y &= -35 \\ y &= -5 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (0, 9)$ into this equation to get the value for $c$.
$9 = c$.
Hence the equation of the line is $y = 9$.

**16.** The original line has an infinite gradient; it is vertical and parallel to the $y$-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(-5, 8)$ lies on the new line, so the equation of the new line is $x = -5$.

**17.** To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -63 &= -7y, \text{ so} \\ 7y &= 63 \\ y &= 9 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(2, -7)$ lies on the new line, so the equation of the new line is $x = 2$.

**18.** The original line has an infinite gradient; it is vertical and parallel to the $y$-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-9, 4)$ lies on the new line, so the equation of the new line is $y = 4$.