MATH 1040 WEEK 12 SOLUTIONS

(1.) $y = \sin(3x^2+4x)$

let $u = 3x^2+4x$, $y = \sin u$

$\frac{du}{dx} = 6x+4$ $\quad \frac{dy}{du} = \cos u$

$\therefore y' = \frac{du}{dx}\cdot\frac{dy}{du} = (6x+4)\cos u$

$= (6x+4)\cos(3x^2+4x)$

(2) $y = (10x^2+10)^3$

let $u = 10x^2+10$, $y = u^3$

$\frac{du}{dx} = 20x$ $\quad \frac{dy}{du} = 3u^2$

$\therefore y' = \frac{dy}{dx} = \frac{du}{dx}\cdot\frac{dy}{du}$

$= 20x(3u^2) = 60x(10x^2+10)$

(3) $y = -2\ln x$

$y' = -\frac{2}{x} = -2x^{-1}$

$y'' = 2x^{-2} \left(= \frac{2}{x^2}\right)$

(4.) $f(x) = -x^3+3x-7$

$f'(x) = -3x^2+3$

Crit. pt when $f'(x) = 0$

$\Rightarrow -3x^2+3 = 0$

$-3(x^2-1) = 0$

$x^2-1 = 0$

$x^2 = 1$

$x = \pm 1$

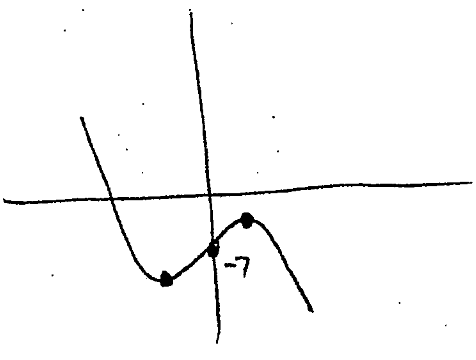
when $x = 1$, $y = -1+3-7 = -5$

$x = -1$, $y = 1-3-7 = -9$

$f''(x) = -6x$

$f''(1) = -6 \quad \therefore$ local max. $\cap$ at $(1,-5)$

$f''(-1) = 6 \quad \therefore$ local min. $\cup$ at $(-1,-9)$



⑤ $f(x) = -x^3+3x-7$

$f'(x) = -3x^2+3$

$f'(-2) = -3\times4+3$

$= -9$

$\therefore$ Eq. of tangent at $(-2,-5)$ is $y=-9x+c$.

$-5 = -9\times-2+c$

$\therefore c = -23$

$\therefore y = -9x-23$

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⑥ $f(t) = 42t-5t^2+5$

a) $v(t) = f'(t) = 42-10t$

b) $v(1.5) = 42-10\times1.5$

$= 27$ m/s

c) Max. height when $v(t)=0$

$\Rightarrow 42-10t=0$

$t = 4.2$ sec

$f(4.2) = 42\times4.2-5\times4.2^2+5$

$= 93.2$ m

d) $a(t) = v'(t) = -10$

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⑦ Let 1st no. $= x$
2nd no. $= y$

So $xy = 48$ — ①

Want to minimise $S(y) = x+y^3$ — ②

From ①, $x = \frac{48}{y}$. Sub into ② $\Rightarrow \frac{48}{y}+y^3 = S(y)$

Crit pt when $S'(y)=0 \Rightarrow -\frac{48}{y^2}+3y^2=0 \Rightarrow 3y^2 = \frac{48}{y^2}$

$\Rightarrow 3y^4=48 \Rightarrow y^4=16 \Rightarrow y=2 \therefore x=24$

$S''(y) = +96/y^3+6y$, $S''(2) = +ve \therefore$ local min. ✓