## MATH1040/7040 Week 11 SOLUTIONS

**1.** $y = -5 + 4x$, so

$$\begin{aligned} y' &= 1 \times 4x^{1-1} \\ &= 4 \end{aligned}$$

**2.** $y = 8x^4 + \dfrac{2}{x^7}$, so $y = 8x^4 + 2x^{-7}$, so

$$\begin{aligned} y' &= 4 \times 8x^{4-1} - 7 \times 2x^{-7-1} \\ &= 32x^3 - 14x^{-8} \\ &= 32x^3 - \frac{14}{x^8} \end{aligned}$$

**3.** $y = -4\sin x + 2\cos x$, so

$$\begin{aligned} y' &= -4\cos x + 2 \times (-\sin x) \\ &= -4\cos x - 2\sin x \end{aligned}$$

**4.** $y = 6\sqrt{x} - 3e^x + 4\cos x$, so $y = 6x^{\frac{1}{2}} - 3e^x + 4\cos x$, so

$$\begin{aligned} y' &= \frac{1}{2} \times 6 \times x^{\frac{1}{2}-1} - 3e^x + 4 \times (-\sin x) \\ &= 3x^{-\frac{1}{2}} - 3e^x - 4\sin x \\ &= \frac{3}{\sqrt{x}} - 3e^x - 4\sin x \end{aligned}$$

Hence $y' = \dfrac{3}{\sqrt{x}} - 3e^x - 4\sin x$.

**5.** Let $u = 8x^2 - 9$, so $y = u^5$.

Now $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$.

$\dfrac{dy}{du} = 5 \times u^{5-1} = 5u^4$

$\dfrac{du}{dx} = 8 \times 2 \times x^{2-1} = 16x$

So, $\dfrac{dy}{dx} = 5u^4 \times 16x = 5\left(8x^2 - 9\right)^4 \times 16x = 80x\left(8x^2 - 9\right)^4$.

Hence $\dfrac{dy}{dx} = 80x\left(8x^2 - 9\right)^4$.

**6.** Let $u = 7x^7 + 4$, so $y = \dfrac{1}{u^8} = u^{-8}$.

Now $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$.

$\dfrac{dy}{du} = -8 \times u^{-8-1} = -8u^{-9}$

$\dfrac{du}{dx} = 7 \times 7 \times x^{7-1} = 49x^6$

So, $\dfrac{dy}{dx} = -8u^{-9} \times 49x^6 = -8\left(7x^7 + 4\right)^{-9} \times 49x^6 = -392x^6\left(7x^7 + 4\right)^{-9} = -\dfrac{392x^6}{\left(7x^7 + 4\right)^9}$.

Hence $\dfrac{dy}{dx} = -\dfrac{392x^6}{\left(7x^7 + 4\right)^9}$.

**7.** Let $u = 3x^7 - 9$, so $y = u^8$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$\frac{dy}{du} = 8 \times u^{8-1} = 8u^7$

$\frac{du}{dx} = 3 \times 7 \times x^{7-1} = 21x^6$

So, $\frac{dy}{dx} = 8u^7 \times 21x^6 = 8\left(3x^7 - 9\right)^7 \times 21x^6 = 168x^6\left(3x^7 - 9\right)^7$.

Hence $\frac{dy}{dx} = 168x^6\left(3x^7 - 9\right)^7$.

**8.** Let $u = 2z^2 - 4z^3$, then $u' = 4z - 12z^2$.
Let $v = 5 + z^2$, then $v' = 2z$.
Product rule: $y' = u'v + uv'$.

Substitute $u, u', v$ and $v'$ into the product rule:

$$\begin{aligned} y' &= \left(4z - 12z^2\right) \times \left(5 + z^2\right) + \left(2z^2 - 4z^3\right) \times 2z \\ &= 20z + 4z^3 - 60z^2 - 12z^4 + 4z^3 - 8z^4 \end{aligned}$$

Hence $y' = -20z^4 + 8z^3 - 60z^2 + 20z$.

**9.** Let $u = 5 + 2x^3$, then $u' = 6x^2$.
Let $v = -9x^2 + 8x^3$, then $v' = -18x + 24x^2$.
Product rule: $y' = u'v + uv'$.

Substitute $u, u', v$ and $v'$ into the product rule:

$$\begin{aligned} y' &= 6x^2 \times \left(-9x^2 + 8x^3\right) + \left(5 + 2x^3\right) \times \left(-18x + 24x^2\right) \\ &= -54x^4 + 48x^5 - 90x + 120x^2 - 36x^4 + 48x^5 \end{aligned}$$

Hence $y' = 96x^5 - 90x^4 + 120x^2 - 90x$.

**10.** Let $u = 8h - 8h^3$, then $u' = 8 - 24h^2$.
Let $v = 9h^3 - 8$, then $v' = 27h^2$.
Product rule: $y' = u'v + uv'$.

Substitute $u, u', v$ and $v'$ into the product rule:

$$\begin{aligned} y' &= \left(8 - 24h^2\right) \times \left(9h^3 - 8\right) + \left(8h - 8h^3\right) \times 27h^2 \\ &= 72h^3 - 64 - 216h^5 + 192h^2 + 216h^3 - 216h^5 \end{aligned}$$

Hence $y' = -432h^5 + 288h^3 + 192h^2 - 64$.

**11.** Q1 $f'(x) = -3x^2 - 24x - 48$

Q2 $f'(x) = 0$, so from Q1 , $-3x^2 - 24x - 48 = 0$, so we use $a = -3, b = -24, c = -48$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{24 \pm \sqrt{(-24)^2 - 4 \times (-3) \times (-48)}}{2 \times (-3)} \\ &= \frac{24 \pm \sqrt{576 - 576}}{-6} \\ &= \frac{24 \pm \sqrt{0}}{-6} \\ &= \frac{24}{-6} \\ &= -4 \end{aligned}$$

Q3 $f''(x) = -6x - 24$

Q4 $f'(-3) = -3 \times (-3)^2 - 24 \times (-3) - 48 = -3$

**12.** Let $u = 4r^2 + 5r$, then $u' = 8r + 5$.
Let $v = -7r^2 + 4r$, then $v' = -14r + 4$.

Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$ , so

$$\begin{aligned} y' &= \frac{(8r+5) \times \left(-7r^2+4r\right) - \left(4r^2+5r\right) \times (-14r+4)}{\left(-7r^2+4r\right)^2} \\ &= \frac{-56r^3 + 32r^2 - 35r^2 + 20r - \left(-56r^3 + 16r^2 - 70r^2 + 20r\right)}{\left(-7r^2+4r\right)^2} \\ &= \frac{-56r^3 + 32r^2 - 35r^2 + 20r + 56r^3 - 16r^2 + 70r^2 - 20r}{\left(-7r^2+4r\right)^2} \end{aligned}$$

Hence $y' = \dfrac{51r^2}{\left(-7r^2+4r\right)^2}$.