1.  

(a) 

\[-5z (-6 + 2z) = -6 \times (-5z) + 2z \times (-5z) = 30z - 10z^2\]

(b) 

\[(-6 + 7x) (3 - 6x) = -6 \times 3 - 6 \times (-6x) + 7x \times 3 + 7x \times (-6x) = -18 + 36x + 21x - 42x^2 = -42x^2 + 57x - 18\]

(c) Substituting for \(z\) into the equation gives \(2 = 4x - 3\), so \(4x = 2 + 3\), so \(4x = 5\), so \(\frac{4x}{4} = \frac{5}{4}\)

Hence \(x = \frac{5}{4}\)

(d) 

\[-4x - 6 = 0\), so \(-4x = 6\), so \(-\frac{4x}{4} = \frac{6}{4}\)

Hence \(x = -\frac{3}{2}\)

(e) 

\[-\frac{4x}{2} - 6 = 2\), so \(-2x = 2 + 6\), so \(-2x = 8\), so \(-\frac{2x}{2} = \frac{8}{2}\)

Hence solution is: \(x = -4\)

(f) 

\[-4 + \frac{-4}{4x} = 2\), so \(\frac{1}{x} = 4 + 2\), so \(\frac{1}{x} = 6\), so \(1 = 6x\), so \(x = \frac{1}{6}\)

Hence solution is: \(x = \frac{1}{6}\)

(g) 

\[-\frac{11}{3} \times \frac{13}{5} = \frac{-11 \times 13}{3 \times 5} = \frac{-143}{15} = -9\frac{8}{15}\]

Hence solution is: \(z = -9\frac{8}{15}\)

(h) 

\(6 = 4z + 4\), so \(6 - 4 = 4z\), so \(2 = 4z\), so \(\frac{2}{4} = \frac{4z}{4}\)

Hence \(z = \frac{1}{2}\)

(i) \(|5x + 5| = 1\), so

\[
5x + 5 = 1 \quad \text{or} \quad 5x + 5 = -1 \\
5x = 1 - 5 \quad 5x = -1 - 5 \\
5x = -4 \quad 5x = -6 \\
\frac{5x}{5} = -4 \quad \frac{5x}{5} = -6 \\
\]

Hence the solutions are: \(x = -\frac{4}{5}\) and \(x = -\frac{6}{5}\)

(j) Since the two integers are consecutive, we know that there is a difference of one between them. Let the smaller integer be represented by \(n\), so the larger integer will then be \((n + 1)\). We then have:
\[ n + (n + 1) = 15 \]
\[ \implies 2 \times n + 1 = 15 \]
\[ \implies 2 \times n = 14 \]
\[ \implies n = 7 \]

Note that this gives us the value of the lower integer only! We need both integers!

So if the smaller number is 7, then the larger number must be 8.

(k) \( \sqrt{128} = 4\sqrt{2} \), so \( 128x = \sqrt{4 \times 4 \times 8} = \sqrt{128} \), so \( 128x = 128 \). Hence \( x = 1 \)

(l) \( \sqrt{45} = x\sqrt{5} \). Now \( \sqrt{45} = \sqrt{9 \times 5} = \sqrt{3 \times 3 \times 5} = 3\sqrt{5} \). Hence \( x = 3 \)

(m) \[
\sqrt{2} \left( \sqrt{5} + \sqrt{4} \right) = \sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{4} \\
= \sqrt{10} + \sqrt{8} \\
= \sqrt{10} + 2\sqrt{2}
\]

(n) \[
\left( \sqrt{6} + \sqrt{6} \right) \left( \sqrt{8} + \sqrt{6} \right) = \sqrt{6} \times \sqrt{8} + \sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{8} + \sqrt{6} \times \sqrt{6} \\
= \sqrt{6} \times \sqrt{8} + \sqrt{6} \times \sqrt{8} + \sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{6} \\
= \sqrt{48} + \sqrt{48} + \sqrt{48} + \sqrt{48} \\
= 4\sqrt{3} + 6 + 4\sqrt{3} + 6 \\
= 6 + 6 + 4\sqrt{3} + 4\sqrt{3} \\
= 12 + 8\sqrt{3}
\]

2.

(a) \[
y^{-2}y^{-3}x^2y^{-3} \div x^{-1} \times y^{-2} = y^{-2}y^{-3}x^2y^{-3} \times x^1 \times y^{-2} \\
= x^2x^1y^{-2}y^{-3}y^{-3}y^{-2} \\
= x^{2+1}y^{-2-3-3-2} \\
= x^3y^{-10}
\]

(b) \[
\frac{-5y^4}{y^{-3}y^{-5}} = \frac{-5y^{4+4}}{y^{-3-5}} = \frac{-5y^8}{y^{-8}} = -5y^{8-(-8)} = -5y^{16}
\]
3.

(a) In interval form the answer is \((-\infty, 6.6)\) and on a real line the answer is:

(b) In inequality form the answer is \(9 \leq x < 11\) and on a real line the answer is:

(c)\[
\begin{align*}
8x - 6 &\geq 4x - 10 \\
8x - 6 + 6 &\geq 4x - 10 + 6 \\
8x &\geq 4x - 4 \\
8x - 4x &\geq 4x - 4x - 4 \\
4x &\geq -4 \\
4x \div 4 &\geq -4 \div 4 \\
x &\geq -1
\end{align*}
\]

In interval format the answer is \([-1, \infty)\), and on a real line the answer is:
4. Mayumi ate $x$ pieces of sushi. Rumi ate 4 more, so $x + 4$.
So, $x + x + 4 = 26$
$2x = 22$
$x = 11$

So Mayumi ate 11 pieces and Rumi ate $11 + 4 = 15$ pieces (check: $11 + 15 = 26$)

5. Let the first hospital have $x$ doctors. The second hospital therefore has $3x - 20$ doctors.
So, $x + 3x - 20 = 204$
$4x = 224$
$x = 56$

So the first hospital has 56 doctors and the second hospital has $56 + 3 \times 56 - 20 = 148$. (check: $56 + 148 = 204$)

6. \[
\frac{(x + x^2) \div x - 16 - x}{3} = \frac{\left(\frac{x + x^2}{x} - 16 - x\right)}{3}
\]
\[
= \left(\frac{x(1 + x)}{x} - 16 - x\right) \div 3
\]
\[
= (1 + x - 16 - x) \div 3
\]
\[
= -15 \div 3
\]
\[
= -5
\]

The $x$’s disappear, so regardless of what number $x$ is the answer is always $-5$. 