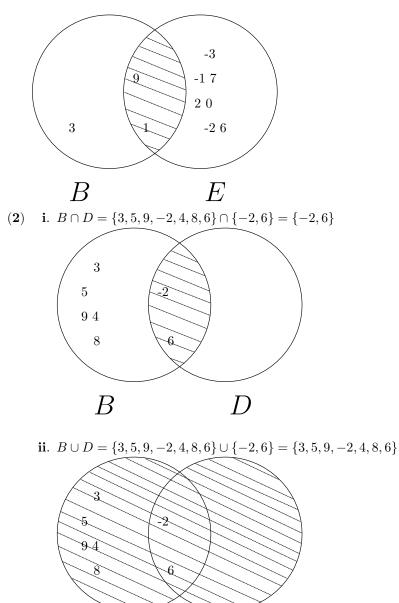
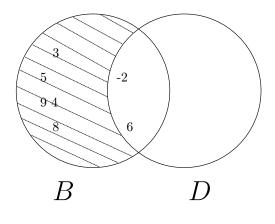
**1.** (1)  $B \cap E = \{3, 9, 1\} \cap \{-3, -1, 7, 2, 0, 9, -2, 1, 6\} = \{9, 1\}$ On Venn diagram:



BD

iii.  $B \setminus D = \{3, 5, 9, -2, 4, 8, 6\} \setminus \{-2, 6\} = \{3, 5, 9, 4, 8\}$ 

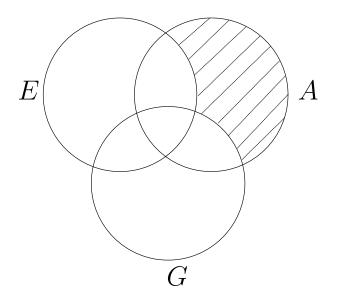


iv.  $D \setminus B = \{-2, 6\} \setminus \{3, 5, 9, -2, 4, 8, 6\} = \emptyset$ 3 5 9 4 8 6 B D

(3) i.  $G = \{3, -1\}$ 

ii.  $G \cup A = \{3, -1\} \cup \{3, 5, 7, 4, 8, 6\} = \{3, -1, 5, 7, 4, 8, 6\}$ iii.  $G \cap A = \{3, -1\} \cap \{3, 5, 7, 4, 8, 6\} = \{3\}$ iv.  $G \setminus A = \{3, -1\} \setminus \{3, 5, 7, 4, 8, 6\} = \{-1\}$ v.  $A \setminus (G \cup E) = \{3, 5, 7, 4, 8, 6\} \setminus (\{3, -1\} \cup \{-2\})$ 

$$A \setminus (G \cup E) = \{3, 5, 7, 4, 8, 6\} \setminus (\{3, -1\} \cup \{-2\})$$
$$= \{3, 5, 7, 4, 8, 6\} \setminus \{3, -1, -2\}$$
$$= \{5, 7, 4, 8, 6\}$$



$$\begin{aligned} (G \cup A) \setminus & E = (\{3, -1\} \cup \{3, 5, 7, 4, 8, 6\}) \setminus \{-2\} \\ &= \{3, -1, 5, 7, 4, 8, 6\} \setminus \{-2\} \\ &= \{3, -1, 5, 7, 4, 8, 6\} \end{aligned}$$

vii.

$$\begin{split} E \cup (G \cup A) &= \{-2\} \cup (\{3, -1\} \cup \{3, 5, 7, 4, 8, 6\}) \\ &= \{-2\} \cup \{3, -1, 5, 7, 4, 8, 6\} \\ &= \{3, 5, -1, 7, -2, 4, 8, 6\} \end{split}$$

**viii**. 
$$E \cap \emptyset = \{-2\} \cap \emptyset = \emptyset$$

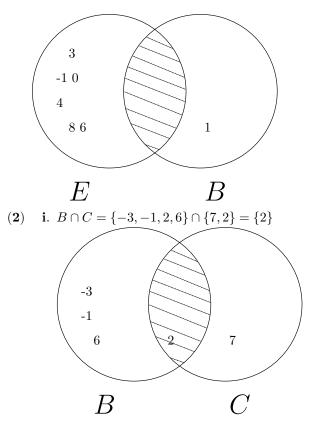
$$(A \cap G) \cup (A \cap E) = (\{3, 5, 7, 4, 8, 6\} \cap \{3, -1\}) \cup (\{3, 5, 7, 4, 8, 6\} \cap \{-2\})$$
$$= \{3\} \cup \emptyset$$
$$= \{3\}$$

(4) i. 
$$Prob(s_1 \text{ is even}) = \frac{3}{6} = \frac{1}{2}$$
  
ii.  $Prob(s_1 = 5) = \frac{1}{6}$   
iii.  $Prob(s_1 < 2) = \frac{1}{6}$   
iv.  $Prob(s_1 \text{ is even and } s_1 < 2) = \frac{0}{6} = 0$   
v.  $Prob(s_1 \text{ is even or } s_1 < 2) = \frac{4}{6} = \frac{2}{3}$ 

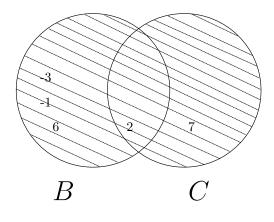
- vi.  $Prob(s_1 \text{ is even given that } s_1 < 2) = \frac{0}{1} = 0$
- **vii**.  $Prob(s_1 \text{ is even}) = \frac{1}{2}$ , and  $Prob(s_2 \text{ is even}) = \frac{1}{3}$ . Now  $s_1$  and  $s_2$  are chosen independently, so  $Prob(both s_1 \text{ and } s_2 \text{ are even}) = Prob(s_1 \text{ is even}) \times Prob(s_2 \text{ is even})$ . Hence  $Prob(both s_1 \text{ and } s_2 \text{ are even}) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
- viii. By the principle of inclusion\exclusion,

 $Prob(s_1 \text{ is even } \mathbf{or} \ s_2 \text{ is even}) = Prob(s_1 \text{ is even}) + Prob(s_2 \text{ is even}) - Prob(both \ s_1 \text{ and } s_2 \text{ are even }).$ Hence  $Prob(s_1 \text{ is even } \mathbf{or} \ s_2 \text{ is even}) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$ 

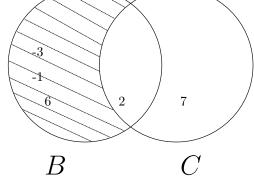
- ix. Now  $s_1$  and  $s_2$  are chosen independently, so  $Prob(s_1 \text{ is even given that } s_2 \text{ is even }) = Prob(s_1 \text{ is even}).$ Hence  $Prob(s_1 \text{ is even given that } s_2 \text{ is even }) = \frac{1}{2}$
- **2.** (1)  $E \cap B = \{3, -1, 0, 4, 8, 6\} \cap \{1\} = \emptyset$ On Venn diagram:

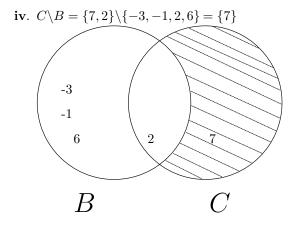


ii.  $B \cup C = \{-3, -1, 2, 6\} \cup \{7, 2\} = \{-3, -1, 7, 2, 6\}$ 

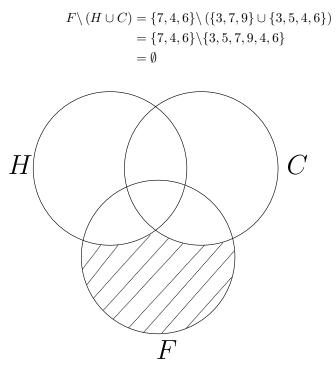


iii.  $B \setminus C = \{-3, -1, 2, 6\} \setminus \{7, 2\} = \{-3, -1, 6\}$ 





- (3) i.  $C = \{3, 5, 4, 6\}$ 
  - ii.  $C \cup F = \{3, 5, 4, 6\} \cup \{7, 4, 6\} = \{3, 5, 7, 4, 6\}$ iii.  $F \cap C = \{7, 4, 6\} \cap \{3, 5, 4, 6\} = \{4, 6\}$ iv.  $C \setminus F = \{3, 5, 4, 6\} \setminus \{7, 4, 6\} = \{3, 5\}$



vi.

$$(F \cap C) \cap H = (\{7, 4, 6\} \cap \{3, 5, 4, 6\}) \cap \{3, 7, 9\}$$
$$= \{4, 6\} \cap \{3, 7, 9\}$$
$$= \emptyset$$

vii.

$$\begin{split} C \cup (F \cap H) &= \{3, 5, 4, 6\} \cup (\{7, 4, 6\} \cap \{3, 7, 9\}) \\ &= \{3, 5, 4, 6\} \cup \{7\} \\ &= \{3, 5, 7, 4, 6\} \end{split}$$

viii. 
$$\emptyset \cap C = \emptyset \cap \{3, 5, 4, 6\} = \emptyset$$

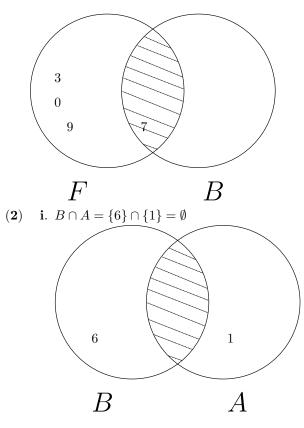
$$(C \cup H) \setminus (F \cup \emptyset) = (\{3, 5, 4, 6\} \cup \{3, 7, 9\}) \setminus (\{7, 4, 6\} \cup \emptyset)$$
$$= \{3, 5, 7, 9, 4, 6\} \setminus \{7, 4, 6\}$$
$$= \{3, 5, 9\}$$

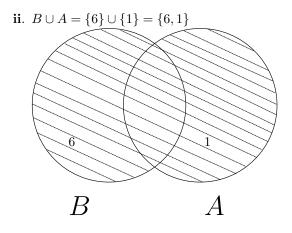
(4) i. 
$$Prob(t_1 \text{ is odd}) = \frac{1}{2}$$
  
ii.  $Prob(t_1 = 2) = \frac{1}{2}$ 

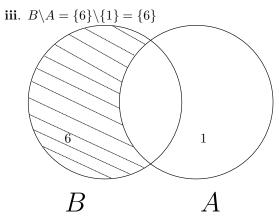
- **iii.**  $Prob(t_1 > 1) = \frac{1}{2}$
- **iv.**  $Prob(t_1 \text{ is odd and } t_1 > 1) = \frac{0}{2} = 0$
- **v**.  $Prob(t_1 \text{ is odd or } t_1 > 1) = \frac{2}{2} = 1$
- vi.  $Prob(t_1 \text{ is odd given that } t_1 > 1) = \frac{0}{1} = 0$
- **vii**.  $Prob(t_1 \text{ is odd}) = \frac{1}{2}$ , and  $Prob(t_2 \text{ is odd}) = \frac{3}{7}$ . Now  $t_1$  and  $t_2$  are chosen independently, so  $Prob(both t_1 \text{ and } t_2 \text{ are odd}) = Prob(t_1 \text{ is odd}) \times Prob(t_2 \text{ is odd})$ . Hence  $Prob(both t_1 \text{ and } t_2 \text{ are odd}) = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$
- viii. By the principle of inclusion/exclusion,

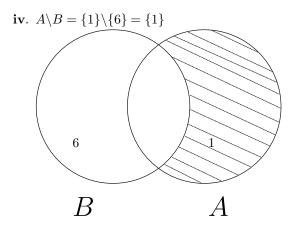
 $Prob(t_1 \text{ is odd } \mathbf{or} \ t_2 \text{ is odd}) = Prob(t_1 \text{ is odd}) + Prob(t_2 \text{ is odd}) - Prob(both \ t_1 \text{ and } t_2 \text{ are odd }).$ Hence  $Prob(t_1 \text{ is odd } \mathbf{or} \ t_2 \text{ is odd}) = \frac{1}{2} + \frac{3}{7} - \frac{3}{14} = \frac{5}{7}$ 

- ix. Now  $t_1$  and  $t_2$  are chosen independently, so  $Prob(t_1 \text{ is odd given that } t_2 \text{ is odd }) = Prob(t_1 \text{ is odd}).$ Hence  $Prob(t_1 \text{ is odd given that } t_2 \text{ is odd }) = \frac{1}{2}$
- **3.** (1)  $F \cap B = \{3, 7, 0, 9\} \cap \{7\} = \{7\}$ On Venn diagram:

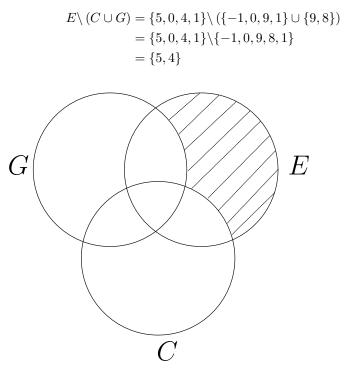








- (3) i.  $C = \{-1, 0, 9, 1\}$ 
  - ii.  $E \cup C = \{5, 0, 4, 1\} \cup \{-1, 0, 9, 1\} = \{5, -1, 0, 9, 4, 1\}$
  - iii.  $C \cap E = \{-1, 0, 9, 1\} \cap \{5, 0, 4, 1\} = \{0, 1\}$
  - iv.  $E \setminus C = \{5, 0, 4, 1\} \setminus \{-1, 0, 9, 1\} = \{5, 4\}$



$$\begin{split} (G \cup C) \cap E &= (\{9,8\} \cup \{-1,0,9,1\}) \cap \{5,0,4,1\} \\ &= \{-1,0,9,8,1\} \cap \{5,0,4,1\} \\ &= \{0,1\} \end{split}$$

vii.

$$\begin{split} C \cup (E \cap G) &= \{-1, 0, 9, 1\} \cup (\{5, 0, 4, 1\} \cap \{9, 8\}) \\ &= \{-1, 0, 9, 1\} \cup \emptyset \\ &= \{-1, 0, 9, 1\} \end{split}$$

viii. 
$$\emptyset \setminus E = \emptyset \setminus \{5, 0, 4, 1\} = \emptyset$$

 $\mathbf{i}\mathbf{x}$ .

$$\begin{aligned} (C \cup E) \setminus (C \cup G) &= (\{-1, 0, 9, 1\} \cup \{5, 0, 4, 1\}) \setminus (\{-1, 0, 9, 1\} \cup \{9, 8\}) \\ &= \{-1, 5, 0, 9, 4, 1\} \setminus \{-1, 0, 9, 8, 1\} \\ &= \{5, 4\} \end{aligned}$$

(4) i. 
$$Prob(s_1 \text{ is even}) = \frac{2}{4} = \frac{1}{2}$$
  
ii.  $Prob(s_1 = 10) = 0$ 

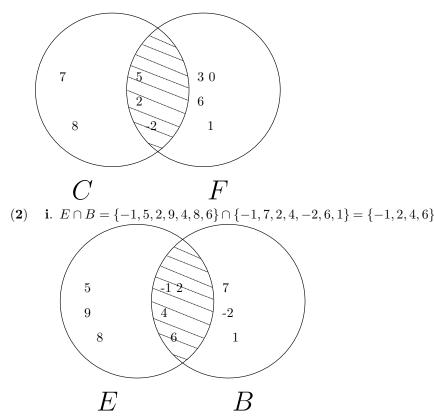
- iii.  $Prob(s_1 < 9) = \frac{3}{4}$
- iv.  $Prob(s_1 \text{ is even and } s_1 < 9) = \frac{2}{4} = \frac{1}{2}$
- **v**.  $Prob(s_1 \text{ is even or } s_1 < 9) = \frac{3}{4}$
- vi.  $Prob(s_1 \text{ is even given that } s_1 < 9) = \frac{2}{3}$
- **vii.**  $Prob(s_1 \text{ is even}) = \frac{1}{2}$ , and  $Prob(s_2 \text{ is even}) = \frac{1}{3}$ . Now  $s_1$  and  $s_2$  are chosen independently,

so Prob (both  $s_1$  and  $s_2$  are even  $) = Prob(s_1 \text{ is even}) \times Prob(s_2 \text{ is even}).$ 

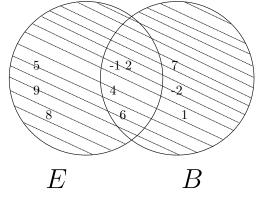
- Hence Prob (both  $s_1$  and  $s_2$  are even  $) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
- viii. By the principle of inclusion/exclusion,

 $Prob(s_1 \text{ is even } \mathbf{or} \ s_2 \text{ is even}) = Prob(s_1 \text{ is even}) + Prob(s_2 \text{ is even}) - Prob(both \ s_1 \text{ and } s_2 \text{ are even }).$ Hence  $Prob(s_1 \text{ is even } \mathbf{or} \ s_2 \text{ is even}) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$ 

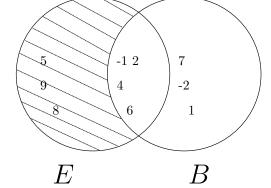
- ix. Now  $s_1$  and  $s_2$  are chosen independently, so  $Prob(s_1 \text{ is even given that } s_2 \text{ is odd}) = Prob(s_1 \text{ is even}).$ Hence  $Prob(s_1 \text{ is even given that } s_2 \text{ is odd}) = \frac{1}{2}$
- 4. (1)  $C \cap F = \{5, 7, 2, -2, 8\} \cap \{3, 5, 2, 0, -2, 1, 6\} = \{5, 2, -2\}$ On Venn diagram:



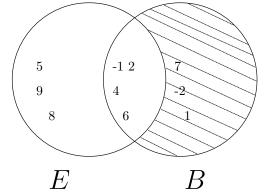
 $\textbf{ii.} \ E \cup B = \{ \underbrace{-1, 5, 2, 9, 4, 8, 6} \} \underbrace{-1, 7, 2, 4, -2, 6, 1} = \{ -1, 5, 7, 2, 9, 4, -2, 8, 6, 1 \}$ 



iii.  $E \setminus B = \{-1, 5, 2, 9, 4, 8, 6\} \setminus \{-1, 7, 2, 4, -2, 6, 1\} = \{5, 9, 8\}$ 



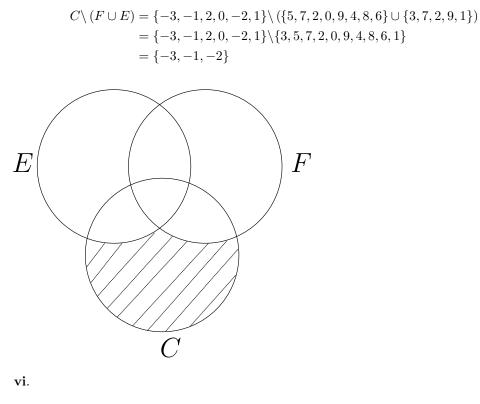
iv.  $B \setminus E = \{-1, 7, 2, 4, -2, 6, 1\} \setminus \{-1, 5, 2, 9, 4, 8, 6\} = \{7, -2, 1\}$ 



(3) i.  $F = \{5, 7, 2, 0, 9, 4, 8, 6\}$ 

ii.  $C \cup E = \{-3, -1, 2, 0, -2, 1\} \cup \{3, 7, 2, 9, 1\} = \{-3, 3, -1, 7, 2, 0, 9, -2, 1\}$ iii.  $C \cap F = \{-3, -1, 2, 0, -2, 1\} \cap \{5, 7, 2, 0, 9, 4, 8, 6\} = \{2, 0\}$ 

iv.  $F \setminus E = \{5, 7, 2, 0, 9, 4, 8, 6\} \setminus \{3, 7, 2, 9, 1\} = \{5, 0, 4, 8, 6\}$ 



$$\begin{split} (F \cup E) \cap C &= (\{5,7,2,0,9,4,8,6\} \cup \{3,7,2,9,1\}) \cap \{-3,-1,2,0,-2,1\} \\ &= \{3,5,7,2,0,9,4,8,6,1\} \cap \{-3,-1,2,0,-2,1\} \\ &= \{2,0,1\} \end{split}$$

vii.

$$\begin{split} F \cup (C \cap E) &= \{5, 7, 2, 0, 9, 4, 8, 6\} \cup (\{-3, -1, 2, 0, -2, 1\} \cap \{3, 7, 2, 9, 1\}) \\ &= \{5, 7, 2, 0, 9, 4, 8, 6\} \cup \{2, 1\} \\ &= \{5, 7, 2, 0, 9, 4, 8, 6, 1\} \end{split}$$

**viii**. 
$$\emptyset \cup C = \emptyset \cup \{-3, -1, 2, 0, -2, 1\} = \{-3, -1, 2, 0, -2, 1\}$$

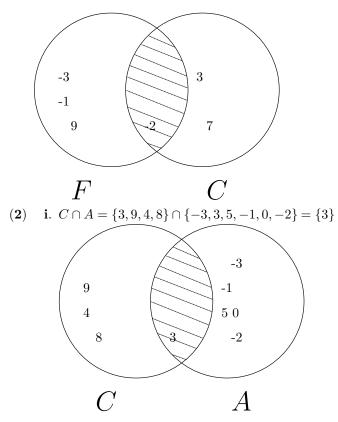
$$(C \cup F) \cap (F \cap E) = (\{-3, -1, 2, 0, -2, 1\} \cup \{5, 7, 2, 0, 9, 4, 8, 6\}) \cap (\{5, 7, 2, 0, 9, 4, 8, 6\} \cap \{3, 7, 2, 9, 1\})$$
$$= \{-1, 7, 2, 0, -2, 1, 6, -3, 5, 9, 4, 8\} \cap \{7, 2, 9\}$$
$$= \{7, 2, 9\}$$

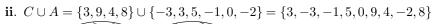
(4) i. 
$$Prob(r_1 \text{ is odd}) = \frac{2}{5}$$
  
ii.  $Prob(r_1 = 6) = \frac{1}{5}$ 

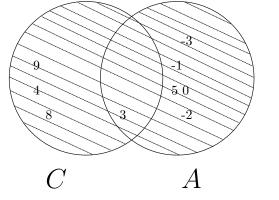
- **iii**.  $Prob(r_1 > 2) = \frac{4}{5}$
- **iv**. *Prob*  $(r_1 \text{ is odd and } r_1 > 2) = \frac{2}{5}$
- **v**. *Prob*  $(r_1 \text{ is odd or } r_1 > 2) = \frac{4}{5}$
- **vi**. *Prob*  $(r_1 \text{ is odd given that } r_1 > 2) = \frac{2}{4} = \frac{1}{2}$
- **vii.**  $Prob(r_1 \text{ is odd}) = \frac{2}{5}$ , and  $Prob(r_2 \text{ is odd}) = \frac{3}{6} = \frac{1}{2}$ . Now  $r_1$  and  $r_2$  are chosen independently, so  $Prob(both r_1 \text{ and } r_2 \text{ are odd}) = Prob(r_1 \text{ is odd}) \times Prob(r_2 \text{ is odd})$ . Hence  $Prob(both r_1 \text{ and } r_2 \text{ are odd}) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$
- **viii**. By the principle of inclusion\exclusion,

 $Prob(r_1 \text{ is odd } \mathbf{or} \ r_2 \text{ is odd}) = Prob(r_1 \text{ is odd}) + Prob(r_2 \text{ is odd}) - Prob(both \ r_1 \text{ and } r_2 \text{ are odd }).$ Hence  $Prob(r_1 \text{ is odd } \mathbf{or} \ r_2 \text{ is odd}) = \frac{2}{5} + \frac{1}{2} - \frac{1}{5} = \frac{7}{10}$ 

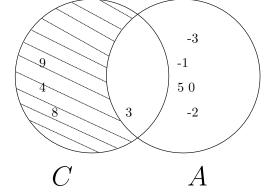
- ix. Now  $r_1$  and  $r_2$  are chosen independently, so  $Prob(r_1 \text{ is odd given that } r_2 \text{ is even }) = Prob(r_1 \text{ is odd}).$ Hence  $Prob(r_1 \text{ is odd given that } r_2 \text{ is even }) = \frac{2}{5}$
- **5.** (1)  $F \cap C = \{-3, -1, 9, -2\} \cap \{3, 7, -2\} = \{-2\}$ On Venn diagram:

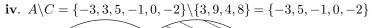


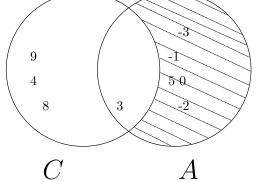




iii.  $C \setminus A = \{3, 9, 4, 8\} \setminus \{-3, 3, 5, -1, 0, -2\} = \{9, 4, 8\}$ 

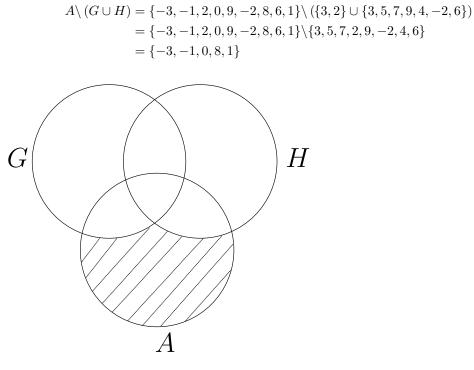






(3) i.  $G = \{3, 2\}$ 

ii.  $G \cup H = \{3, 2\} \cup \{3, 5, 7, 9, 4, -2, 6\} = \{3, 5, 7, 2, 9, -2, 4, 6\}$ iii.  $A \cap H = \{-3, -1, 2, 0, 9, -2, 8, 6, 1\} \cap \{3, 5, 7, 9, 4, -2, 6\} = \{9, -2, 6\}$ iv.  $A \setminus H = \{-3, -1, 2, 0, 9, -2, 8, 6, 1\} \setminus \{3, 5, 7, 9, 4, -2, 6\} = \{-3, -1, 2, 0, 8, 1\}$ 



vi.

$$\begin{split} (A \backslash H) \backslash G &= (\{-3, -1, 2, 0, 9, -2, 8, 6, 1\} \backslash \{3, 5, 7, 9, 4, -2, 6\}) \backslash \{3, 2\} \\ &= \{-3, -1, 2, 0, 8, 1\} \backslash \{3, 2\} \\ &= \{-3, -1, 0, 8, 1\} \end{split}$$

vii.

$$G \setminus (A \setminus H) = \{3, 2\} \setminus (\{-3, -1, 2, 0, 9, -2, 8, 6, 1\} \setminus \{3, 5, 7, 9, 4, -2, 6\})$$
  
=  $\{3, 2\} \setminus \{-3, -1, 2, 0, 8, 1\}$   
=  $\{3\}$ 

**viii**. 
$$H \cup \emptyset = \{3, 5, 7, 9, -2, 4, 6\} \cup \emptyset = \{3, 5, 7, 9, -2, 4, 6\}$$

$$\begin{split} (\emptyset \cup H) \cup (G \cup A) &= (\emptyset \cup \{3, 5, 7, 9, -2, 4, 6\}) \cup (\{3, 2\} \cup \{-3, -1, 2, 0, 9, -2, 8, 1, 6\}) \\ &= \{3, 5, 7, 9, 4, -2, 6\} \cup \{3, -3, -1, 2, 0, 9, -2, 8, 6, 1\} \\ &= \{3, -1, 7, 2, 0, -2, 1, 6, -3, 5, 9, 4, 8\} \end{split}$$

(4) i. 
$$Prob(r_1 \text{ is odd}) = \frac{3}{5}$$
  
ii.  $Prob(r_1 = 9) = \frac{1}{5}$ 

- **iii**.  $Prob(r_1 \ge 6) = \frac{4}{5}$
- iv.  $Prob(r_1 \text{ is odd and } r_1 \ge 6) = \frac{2}{5}$
- **v**. *Prob*  $(r_1 \text{ is odd or } r_1 \ge 6) = \frac{5}{5} = 1$
- vi. Prob  $(r_1 \text{ is odd given that } r_1 \ge 6) = \frac{2}{4} = \frac{1}{2}$
- **vii**.  $Prob(r_1 \text{ is odd}) = \frac{3}{5}$ , and  $Prob(r_2 \text{ is odd}) = \frac{2}{4} = \frac{1}{2}$ . Now  $r_1$  and  $r_2$  are chosen independently, so  $Prob(both r_1 \text{ and } r_2 \text{ are odd}) = Prob(r_1 \text{ is odd}) \times Prob(r_2 \text{ is odd})$ . Hence  $Prob(both r_1 \text{ and } r_2 \text{ are odd}) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$
- viii. By the principle of inclusion\exclusion,

 $Prob(r_1 \text{ is odd } \mathbf{or} \ r_2 \text{ is odd}) = Prob(r_1 \text{ is odd}) + Prob(r_2 \text{ is odd}) - Prob(both \ r_1 \text{ and } r_2 \text{ are odd }).$ Hence  $Prob(r_1 \text{ is odd } \mathbf{or} \ r_2 \text{ is odd}) = \frac{3}{5} + \frac{1}{2} - \frac{3}{10} = \frac{4}{5}$ 

ix. Now  $r_1$  and  $r_2$  are chosen independently, so  $Prob(r_1 \text{ is odd given that } r_2 \text{ is even }) = Prob(r_1 \text{ is odd}).$ Hence  $Prob(r_1 \text{ is odd given that } r_2 \text{ is even }) = \frac{3}{5}$