Math 1040 Basic Mathematics **Practice Problems 3 SOLUTIONS**

1. (**1**) $\dfrac{13z^{-3}z^2}{z^3z^{-3}} = \dfrac{13z^{-3+2}}{z^{3-3}} = \dfrac{13z^{-1}}{z^0} = 13z^{-1-0} = 13z^{-1}$

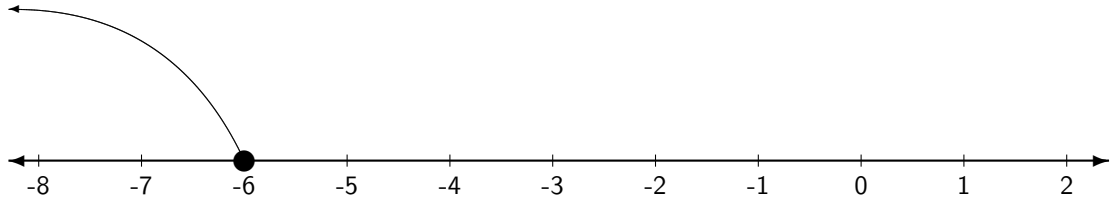
(**2**)

$$\begin{aligned} y^3x^1y^{-1}y^{-3} \div y^3 \times y^{-3} &= y^3x^1y^{-1}y^{-3} \times y^{-3} \times y^{-3} \\ &= x^1y^3y^{-1}y^{-3}y^{-3}y^{-3} \\ &= x^1y^{3-1-3-3-3} \\ &= x^1y^{-7} \\ &= xy^{-7} \end{aligned}$$

(**3**)

$$\begin{aligned} x - 3 &\leq -4x - 33 \\ x - 3 + 3 &\leq -4x - 33 + 3 \\ x &\leq -4x - 30 \\ x + 4x &\leq -4x + 4x - 30 \\ 5x &\leq -30 \\ 5x \div 5 &\leq -30 \div 5 \\ x &\leq -6 \end{aligned}$$

In interval format the answer is $(\infty, -6]$, and on a real line the answer is:



(**4**) $\displaystyle\sum_{i=-1}^{5} -2iz = 2z + 0 - 2z - 4z - 6z - 8z - 10z = -28z$

(**5**) $\displaystyle\sum_{k=1}^{5} (-1)^k\, k = (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 = -1 + 2 - 3 + 4 - 5 = -3$

(**6**) $\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5} = \displaystyle\sum_{k=3}^{5} \frac{1}{k}$

(**7**) $\displaystyle\sum_{k=3}^{6} 2k^3 = 2 \times 3^3 + 2 \times 4^3 + 2 \times 5^3 + 2 \times 6^3 = 54 + 128 + 250 + 432 = 864$

Hence z=864

(**8**) $\displaystyle\sum_{i=-3}^{-2} -x = 8, \quad \text{so} \quad -x - x = 8, \quad \text{so} \quad -2x = 8$

Hence $x = -4$

(**9**) $\sum_{i=4}^{8} yi = 120, \quad$ so $\quad 4y + 5y + 6y + 7y + 8y = 120, \quad$ so $\quad 30y = 120$

Hence $y = 4$

(**10**)

$$\begin{aligned} \sum_{i=z-1}^{z} -4i &= -52 \\ -4(z-1) - 4z &= -52 \\ -8z + 4 &= -52 \\ -8z &= -56 \end{aligned}$$

Hence $z = 7$

2. (**1**) $\dfrac{7x^5x^0}{x^0x^{-5}} = \dfrac{7x^{5+0}}{x^{0-5}} = \dfrac{7x^5}{x^{-5}} = 7x^{5-(-5)} = 7x^{10}$

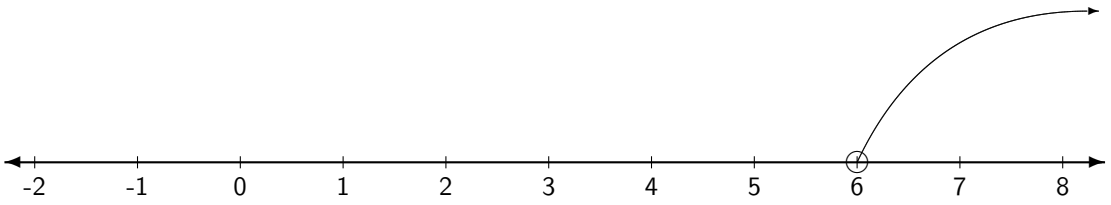
(**2**)

$$\begin{aligned} x^3x^{-1}x^{-1} \times x^{-2}y^{-1} \div y^{-3} &= x^3x^{-1}x^{-1} \times x^{-2}y^{-1} \times y^3 \\ &= x^3x^{-1}x^{-1}x^{-2}y^{-1}y^3 \\ &= x^{3-1-1-2}y^{-1+3} \\ &= x^{-1}y^2 \end{aligned}$$

(**3**)

$$\begin{aligned} -10x - 5 &< -5x - 35 \\ -10x - 5 + 5 &< -5x - 35 + 5 \\ -10x &< -5x - 30 \\ -10x + 5x &< -5x + 5x - 30 \\ -5x &< -30 \\ -5x \div (-5) &> -30 \div (-5) \\ x &> 6 \end{aligned}$$

In interval format the answer is $(6, \infty)$, and on a real line the answer is:



(**4**) $\sum_{i=0}^{2} 4iy = 0 + 4y + 8y = 12y$

(**5**) $\sum_{i=1}^{4} (-2)^i i = (-2)^1 \times 1 + (-2)^2 \times 2 + (-2)^3 \times 3 + (-2)^4 \times 4 = -2 + 8 - 24 + 64 = 46$

(**6**) $3+4+5+6+7=\sum_{j=3}^{7} j$

(**7**) $\sum_{j=-2}^{0} j^0 = (-2)^0 + (-1)^0 + 0^0 = 1+1+1 = 3$

Hence z=3

(**8**) $\sum_{i=-4}^{0} 2y = 40,$ so $2y+2y+2y+2y+2y = 40,$ so $10y = 40$

Hence $y = 4$

(**9**) $\sum_{i=-3}^{-2} yi = -15,$ so $-3y-2y = -15,$ so $-5y = -15$

Hence $y = 3$

(**10**)

$$\sum_{i=x}^{x+2} 2i = 12$$
$$2x+2(x+1)+2(x+2) = 12$$
$$6x+6 = 12$$
$$6x = 6$$

Hence $x = 1$

3. (**1**) $\frac{-7x^{-2}x^2}{x^{-3}x^5} = \frac{-7x^{-2+2}}{x^{-3+5}} = \frac{-7x^0}{x^2} = -7x^{0-2} = -7x^{-2}$

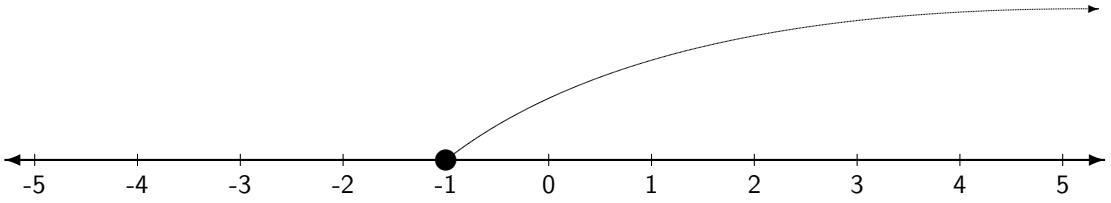
(**2**)

$$\begin{aligned} y^3y^0 \times y^{-3} \div \left(x^{-3}x^2y^3\right) &= y^3y^0 \times y^{-3} \times x^3x^{-2}y^{-3} \\ &= x^3x^{-2}y^3y^0y^{-3}y^{-3} \\ &= x^{3-2}y^{3+0-3-3} \\ &= x^1y^{-3} \\ &= xy^{-3} \end{aligned}$$

(**3**)

$$\begin{aligned} 5x-5 &\geq 4x-6 \\ 5x-5+5 &\geq 4x-6+5 \\ 5x &\geq 4x-1 \\ 5x-4x &\geq 4x-4x-1 \\ x &\geq -1 \end{aligned}$$

In interval format the answer is $[-1,\infty)$, and on a real line the answer is:



(**4**) $\sum_{k=3}^{4} -2kx = -6x - 8x = -14x$

(**5**) $\sum_{j=2}^{6} (-1)^j j = (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 + (-1)^6 \times 6 = 2 - 3 + 4 - 5 + 6 = 4$

(**6**) $\frac{2}{5} + \frac{2}{6} + \frac{2}{7} + \frac{2}{8} = \sum_{j=5}^{8} \frac{2}{j}$

(**7**) $\sum_{j=4}^{6} 4j^0 = 4 \times 4^0 + 4 \times 5^0 + 4 \times 6^0 = 4 + 4 + 4 = 12$

Hence z=12

(**8**) $\sum_{i=-1}^{3} z = -20,$ so $z + z + z + z + z = -20,$ so $5z = -20$

Hence $z = -4$

(**9**) $\sum_{i=-4}^{1} zi = -27,$ so $-4z - 3z - 2z - z + 0 + z = -27,$ so $-9z = -27$

Hence $z = 3$

(**10**)

$$\sum_{k=y-1}^{y} -4k = -20$$

$$-4(y-1) - 4y = -20$$

$$-8y + 4 = -20$$

$$-8y = -24$$

Hence $y = 3$

4. (**1**) $\frac{9y^{-5}y^{-5}}{y^{-2}y^2} = \frac{9y^{-5-5}}{y^{-2+2}} = \frac{9y^{-10}}{y^0} = 9y^{-10-0} = 9y^{-10}$

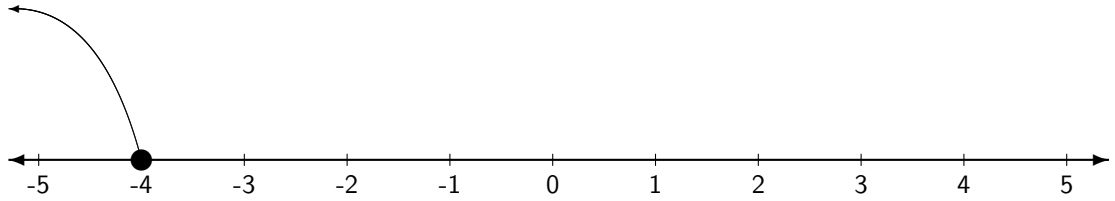
(**2**)

$$\begin{aligned} x^1x^{-3}x^2 \times y^3x^1 \div y^0 &= x^1x^{-3}x^2 \times y^3x^1 \times y^0 \\ &= x^1x^{-3}x^2x^1y^3y^0 \\ &= x^{1-3+2+1}y^{3+0} \\ &= x^1y^3 \\ &= xy^3 \end{aligned}$$

(**3**)

$$\begin{aligned} 5x - 3 &\geq 6x + 1 \\ 5x - 3 + 3 &\geq 6x + 1 + 3 \\ 5x &\geq 6x + 4 \\ 5x - 6x &\geq 6x - 6x + 4 \\ -x &\geq 4 \\ -x \div (-1) &\leq 4 \div (-1) \\ x &\leq -4 \end{aligned}$$

In interval format the answer is $(\infty, -4]$, and on a real line the answer is:



(**4**) $\sum_{k=-5}^{6} ky = -5y - 4y - 3y - 2y - y + 0 + y + 2y + 3y + 4y + 5y + 6y = 6y$

(**5**) $\sum_{j=1}^{4} (-1)^j \, j = (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 = -1 + 2 - 3 + 4 = 2$

(**6**) $-15 - 18 - 21 - 24 - 27$

$= -3 \times 5 - 3 \times 6 - 3 \times 7 - 3 \times 8 - 3 \times 9$

$= \sum_{j=5}^{9} -3j$

(**7**) $\sum_{i=-2}^{2} 2i^3 = 2 \times (-2)^3 + 2 \times (-1)^3 + 2 \times 0^3 + 2 \times 1^3 + 2 \times 2^3 = -16 - 2 + 0 + 2 + 16 = 0$

Hence z=0

(**8**) $\sum_{i=-3}^{1} 3z = 0, \quad$ so $\quad 3z + 3z + 3z + 3z + 3z = 0, \quad$ so $\quad 15z = 0$

Hence $z = 0$

(**9**) $\sum_{i=0}^{2} zi = -3, \quad$ so $\quad 0 + z + 2z = -3, \quad$ so $\quad 3z = -3$

Hence $z = -1$

(**10**)

$$\begin{aligned} \sum_{j=y-1}^{y} -4j &= 4 \\ -4(y-1) - 4y &= 4 \\ -8y + 4 &= 4 \\ -8y &= 0 \end{aligned}$$

Hence $y = 0$

5. (**1**) $\dfrac{9x^{-4}x^1}{x^1x^3} = \dfrac{9x^{-4+1}}{x^{1+3}} = \dfrac{9x^{-3}}{x^4} = 9x^{-3-4} = 9x^{-7}$

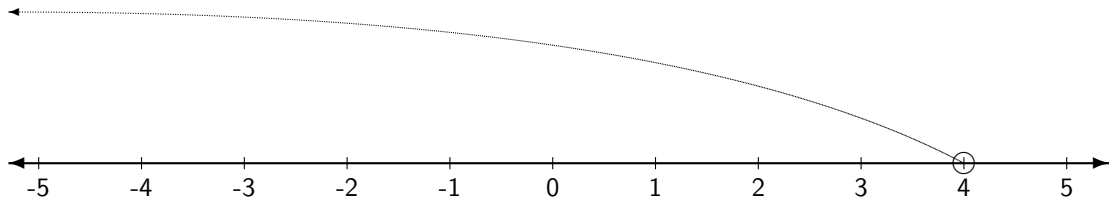
(**2**)

$$\begin{aligned} y^{-1}y^1x^3y^{-1} \times y^0 \div x^{-3} &= y^{-1}y^1x^3y^{-1} \times y^0 \times x^3 \\ &= x^3x^3y^{-1}y^1y^{-1}y^0 \\ &= x^{3+3}y^{-1+1-1+0} \\ &= x^6y^{-1} \end{aligned}$$

(**3**)

$$\begin{aligned} 4x &< -2x + 24 \\ 4x + 2x &< -2x + 2x + 24 \\ 6x &< 24 \\ 6x \div 6 &< 24 \div 6 \\ x &< 4 \end{aligned}$$

In interval format the answer is $(\infty, 4)$, and on a real line the answer is:



(**4**) $\displaystyle\sum_{k=-5}^{-1} 3kx = -15x - 12x - 9x - 6x - 3x = -45x$

(**5**) $\displaystyle\sum_{j=1}^{2} (-1)^j\, j = (-1)^1 \times 1 + (-1)^2 \times 2 = -1 + 2 = 1$

(**6**) $-\dfrac{5}{4} - \dfrac{5}{5} - \dfrac{5}{6} - \dfrac{5}{7} - \dfrac{5}{8} = \displaystyle\sum_{k=4}^{8} \frac{-5}{k}$

(**7**) $\displaystyle\sum_{i=-1}^{1} 2i^1 = 2 \times (-1)^1 + 2 \times 0^1 + 2 \times 1^1 = -2 + 0 + 2 = 0$

Hence y=0

(**8**) $\displaystyle\sum_{i=-3}^{2} 4z = 72,$ so $4z + 4z + 4z + 4z + 4z + 4z = 72,$ so $24z = 72$

Hence $z = 3$

(**9**) $\displaystyle\sum_{i=1}^{2} zi = 6,$ so $z + 2z = 6,$ so $3z = 6$

Hence $z = 2$

(**10**)

$$\sum_{k=y-1}^{y+1} 2k = 18$$

$$2(y-1) + 2y + 2(y+1) = 18$$

$$6y = 18$$

Hence $y = 3$