MATH1040/7040 Assignment 4 SOLUTIONS

1. Answer each of the following questions, showing all working.

(a) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -y + 8 + x &= 6y - 2 - 10x, \text{ so} \\ -y - 6y &= -10x - x - 2 - 8 \\ -7y &= -11x - 10 \\ y &= \frac{11}{7}x + \frac{10}{7} \end{aligned}$$

Hence the gradient is $m = \frac{11}{7}$ and the y-intercept is $c = \frac{10}{7}$.

(b) Thus the equation of the line is $y = -2x+c$ and we can substitute the coordinates of the point $(x_1, y_1) = (6, -10)$ into this equation to get the value for c. Hence $-10 = -2 \times 6 + c$, so $2 = c$.
Hence the equation of the line is $y = -2x + 2$.

(c) Let $(x_1, y_1) = (-9, -4)$ and $(x_2, y_2) = (6, -1)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{6-(-9)} = \frac{3}{15}$. Hence $m = \frac{1}{5}$.

Thus the equation of the line is $y = \frac{1}{5}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-9, -4)$ into this equation to get the value for c.
Hence $-4 = \frac{1}{5} \times (-9) + c$, so $-4 = -\frac{9}{5} + c$. Hence $c = -4 - \left(-\frac{9}{5}\right) = -\frac{11}{5}$.
Hence the equation of the line is $y = \frac{1}{5}x - \frac{11}{5}$.

(d) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 11y - 2x - 5 &= -10 + 10y, \text{ so} \\ 11y - 10y &= 2x - 10 + 5 \\ y &= 2x - 5 \end{aligned}$$

Hence, the gradient of the original line is $m = 2$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = 2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (8, 15)$ into this equation to get the value for c.
$15 = 2 \times 8 + c$, so $15 = 16 + c$. Hence $c = 15 - 16 = -1$.
Hence the equation of the line is $y = 2x - 1$.

(e) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(-7, 1)$ lies on the new line, so the equation of the new line is $x = -7$.

(f) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 16x + 56 &= 8y, \text{ so} \\ -8y &= -16x - 56 \\ y &= 2x + 7 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 2$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\dfrac{1}{m_0}$. Hence $m = -\dfrac{1}{2}$.
Thus the equation of the line is $y = -\dfrac{1}{2}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (20, -11)$ into this equation to get the value of c:
$-11 = -\dfrac{1}{2} \times 20 + c$, so $-11 = -10 + c$. Hence $c = -11 - (-10) = -1$.
Hence the equation of the line is $y = -\dfrac{1}{2}x - 1$.

(g) To determine whether the given line passes through the point $(x_1, y_1) = (-6, 4)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} 0 &= 80 + 24x - 8y, \text{ so} \\ 0 &= 80 + 24 \times (-6) - 8 \times 4 \\ 0 &= 80 - 144 - 32 \\ 0 &= -96 \end{aligned}$$

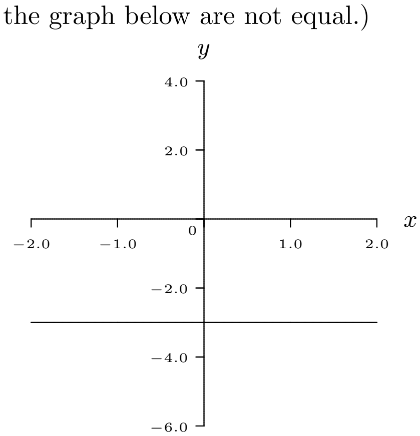
The last statement is **not true**, so our line **does not** pass through the point $(-6, 4)$.

(h) Let $(x_1, y_1) = (\sqrt{20}, -6)$ and $(x_2, y_2) = (\sqrt{5}, -6)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{\left(\sqrt{20} - \sqrt{5}\right)^2 + (-6 - (-6))^2} = \sqrt{\left(\sqrt{4 \times 5} - \sqrt{5}\right)^2 + 0^2}$
$= \sqrt{\left(2\sqrt{5} - \sqrt{5}\right)^2 + 0^2} = \sqrt{5 + 0} = \sqrt{5}$.
Hence $d = \sqrt{5}$

2. **(a)** First we rearrange the equation to get $y = -3$. The line $y = -3$ has constant y-value. Hence, the y-intercept is $y = -3$.

(b) For the line $y = -3$, regardless of the value of x, y is -3. Hence, the line does not intercept the x-axis at all and there is no x-intercept.

(c) (Note that the scaling of the axes on the graph below are not equal.)



3. **(a)** We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 14x + 8y &= -168 \qquad (1) \\ -5x - 3y &= 61 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 and equation (2) by 8 , giving

$$\begin{aligned} 42x + 24y &= -504 \qquad (3) \\ -40x - 24y &= 488 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-40x + 42x - 24y + 24y = 488 - 504 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} 2x &= -16 \qquad (6) \\ x &= -8 \qquad (7) \end{aligned}$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$\begin{aligned} 14 \times (-8) + 8y &= -168 \\ 8y &= -56 \qquad \text{so} \\ y &= -7 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(-8, -7)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\quad 14 \times (-8) + 8 \times (-7) &= -168 \\ -112 - 56 &= -168 \\ -168 &= -168 \end{aligned}$$

$$\begin{aligned} (2)\quad -5 \times (-8) - 3 \times (-7) &= 61 \\ 40 + 21 &= 61 \\ 61 &= 61 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(b) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 81x &= -63 + 9y \qquad (1) \\ -90x - 70 + 10y &= 0 \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$10y = 90x + 70 \qquad (3)$$

Dividing both sides of (3) by 10, gives

$$y = 9x + 7 \qquad (4)$$

Substituting for y in equation (1),

$$81x = -63 + 9 \times (9x + 7) \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} 81x &= -63 + 81x + 63 \\ 0 &= 0 \end{aligned}$$

This statement is **always true**, so there is an infinite number of solutions to our simultaneous equations. The lines are superimposed.

4. Let g =goal and b =behind. North Melbourne's equation is $15g + 14b = 104$ while Melbourne's equation is $12g + 6b = 78$. We could use substitution or elimination to solve for g and b. Elimination is probably easier so let's do that first.

$15g + 14b = 104$ (1)
$12g + 6b = 78$ (2)

(1) $\times$ 3
$45g + 42b = 312$ (3)
(2) $\times$ 7
$84g + 42b = 546$ (4)

(4) $-$ (3)
$39g = 234 \Rightarrow g = 6 \Rightarrow b = 1$
So a goal is worth 6 points and a behind 1. Check this by substituting these values into the original equations.

The numbers in Melbourne's equation are all multiples of 6, so substitution would be pretty quick.
Divide $12g+6b = 78$ by 6 and you get $2g+b = 13$. This means $b = 13-2g$. Substitute this into North Melbourne's equation and you get $15g + 14 \times (13 - 2g) = 104$
So $15g + 182 - 28g = 104$
So $-13g = 104 - 182$
So $g = 6$

5. Any method is fine provided working is shown. No working but correct answer is only worth 1 mark. The answer is 3 diamonds.