10.3 Compound Interest

Compound interest is a way of calculating interest so that you gain interest on interest already gained; that is, it compounds! There are two important formulas for compound interest: compound interest for a particular compounding period and continuously compounding interest.

Compound interest is usually calculated using a specific compounding period. The formula for this is:

$$F(x) = P(1+r)^x$$

- $F(x)$ is the final value of the money
- P is the original amount that we invest
- r is the interest rate per compounding time period (as a decimal)
- x is the number of compounding time periods for which we invest

For example, take an investment of \$100 for 4 years earning 6.75% per annum, compounding every month. Firstly, since the compounding time period is monthly, we need to find how many months in 4 years and what the interest rate per month is. There are $4 \times 12 = 48$ months in 4 years, so we are investing for 48 compounding time periods. The interest rate per annum (per year) is 6.75%, so dividing that by 12 will give the monthly rate: $0.0675 \div 12 = 0.005625$. Now that we have P, r and x we can substitute them into the formula and find the final value.

$$\begin{aligned} F(48) &= 100(1+0.005625)^{48} \\ &= 130.90 \end{aligned}$$

Practice Question 1

If I invest \$250 compounding quarterly at 5.7% per annum for 3 years, how much will I have at the end of the 3 years?

When calculating compound interest with specific compounding time periods, make sure that you convert the interest rate and the total time invested into the **same** units as the compounding period. So, if you are compounding monthly, everything has to be measured in months. If you are compounding weekly then everything has to be measured in weeks.

Sometimes you will have to calculate continuously compounding interest. This is when the interest is being continually added to the account all of the time. For this formula we use an exponential function. It looks like:

$$F(x) = Pe^{rx}$$

- $F(x)$ is the final value of the money
- P is the original amount that we invest
- r is the interest rate per annum (as a decimal)
- x is the time invested in years

Note for continuously compounding interest the time periods are in years.

Again, this is as simple as picking out the correct values from the question and substituting them into the formula. Say I had \$400 invested for $3\frac{1}{2}$ years at 4.5% per annum. What is the final value of my investment after the $3\frac{1}{2}$ years?

$$\begin{aligned} F(3.5) &= 400\exp^{0.045\times 3.5} \\ &= 400\exp^{0.1575} \\ &= 468.23 \end{aligned}$$

Practice Question 2
If I invest \$625 compounding continuously at 7.8% per annum for 5 years, how much will I have at the end of the 5 years?

Discussion Questions
Work through these problems with the person next to you or in a small group.

1. Calculate the final value of an investment that started at \$400 and grew at 4.3% per annum compounding weekly for $2\frac{1}{2}$ years.

2. If I invested \$23 for 100 years compounding quarterly at 5.9% per annum, how much it will be at the end of the 100 years?

3. Calculate the final value of an investment of \$43 at 8.45% per annum compounding annually for 4 years and 3 months.

4. If I invested \$450 for 6 years and 7 months compounding continuously at 8.12% per annumn, how much would I receive at the end of the investment?

5. Your grandmother gives you \$2000 for your excellent results in MATH1040 and you decide to invest it until you graduate university in 3 years' time. You look around the banks and find three options. The Big Red Car Bank offers you 6% interest per annum compounding yearly. The Little Blue Tractor Bank offers 5.89% per annum compoundly quarterly. The Yellow Submarine Bank offers 5.8% per annum compounding continuously. Which of the three banks gives a better return?