9 Quadratic Equations

Similarly to how we needed to be able to identify the shape of linear equations, we also need to be able to identify the shape of quadratic equations. Quadratics, sometimes called parabolas, are curved lines that look a little like a cup.

The general formula for a quadratic equation is: $y = ax^2 + bx + c$. Each of the values of a, b and c tell us a little about the positioning and shape of the equation.

What does a tell us?

- If a is
  - positive: graph is a valley or a happy face.
  - negative: graph is a hill or a sad face.
- The larger a is, the steeper the graph will be

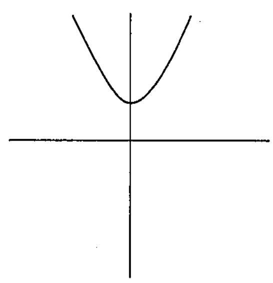
What does c tell us?

- As with linear equations (chapter 6), c is the *y-intercept*, i.e. where the graph *crosses the y-axis.*

The value b is a little tricky; it tells us whether the curve has moved to the left or to the right. We don't really look at b in MATH1040; instead we look at the roots.

The roots of an equation are where the curve *crosses the x-axis.* We find the roots of an equation by solving it. For quadratics you can do this either by factorisation or the quadratic formula. Once you have the roots you can plot them on your graph.

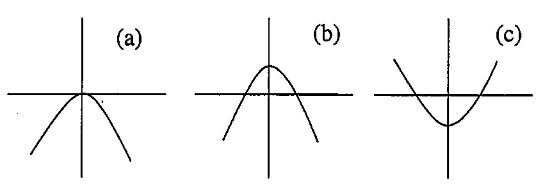
Using the information from a, c and the roots of the equation we can easily sketch most quadratic equations you'll come across. Let's start with a simple equation, $y = 3x^2 + 5$. From the equation we know that it is a valley because a is positive. We also know that it has a y-intercept of 5. So, putting that all together, the equation will look like this:



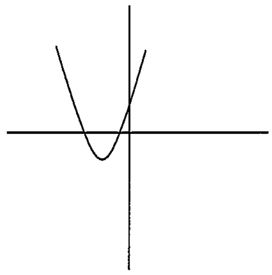
Practice Question 1

Match these 3 quadratic equations to their graphs:

1. $y = 2x^2 - 2$ 2. $y = -3x^2 + 4$ 3. $y = -x^2$

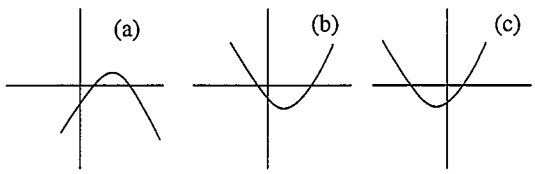


Let's now try one using the roots. Take the equation $y = x^2 + 3x + 2$. In your lecture notes this equation is solved using the quadratic formula giving the roots $x = -1$ and $x = -2$. We also know from the equation that the curve is going to be a valley and the y-intercept is 2. First we plot the three points we have, the roots and y-intercept. Next we join them up with a smooth curve remembering whether it is a valley or hill that we are trying to draw. The x-coordinate of the top of the hill or bottom of the valley will always occur exactly half-way between the two roots (if you have two roots). If you have only one root, that point will be the top of the hill or bottom of the valley. If you have no roots you can't use this method to draw the equation. So plotting our points, and drawing a curve in as a valley we obtain:



Practice Question 2

Which graph represents $y = x^2 + 2x - 8$?



Remember that you may need to expand the equation or rearrange it to equal 0 before you look at the values for a and c.

Discussion Questions

Work through these problems with the person next to you or in a small group.

Match the following quadratic equations with the graphs below:

1. $y = 2x^2$
2. $y = -3x^2 - 5$
3. $x^2 = 2y - 4$
4. $y = x^2 + 2x - 3$
5. $y = (x-1)(x-3)$
6. $y - 15 = 8x + x^2$
7. $y = -x^2 - x + 6$