6 Straight lines and their graphs

It's important to be able to match linear (straight line) equations with their graphs. We often begin with an equation that doesn't look like the standard $y = mx + c$. This means that we need to rearrange the equation so that we can read off from the equation the gradient, m, and the y-intercept, c, in order to visualise the graph. Rearranging equations (called *transposing*) is covered in section 2.3 of the lecture notes. Once we have transposed the equation we then want to determine whether the gradient is positive or negative, and whether the y-intercept is positive or negative. A **positive** gradient means the line slopes up from left to right (a rising line), a **negative** gradient means the line will slope down from left to right (a falling line). A positive y-intercept means the line will cross the y-axis **above** the x-axis, a negative y-intercept means the line will cross the y-axis **below** the x-axis. If there is no c, that is the y-intercept is 0, the line passes through the origin. Using these two values we can visualise any linear equation.

Take for example the equation

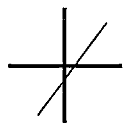
$$2y - 6x = -10$$

The first step is to transpose the equation so it looks like $y = mx + c$.

$$2y = 6x - 10$$

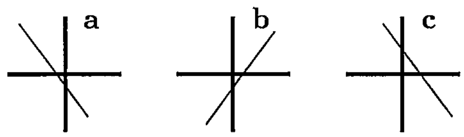
$$y = 3x - 5$$

Next we look at the values for the gradient, m, and the y-intercept, c. The gradient is 3, that means we have a positive gradient, so the line will slope up from left to right. The y-intercept is -5, that means the line will cross below the x-axis. So our equation will roughly look like:

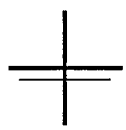


Practice Question 1

Which of the graphs below matches the equation $-\frac{1}{2}y - x = -3$?

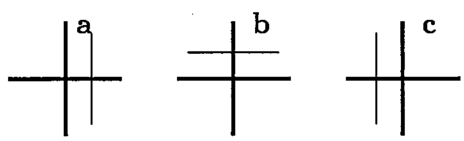


Lines parallel to the axes can look more difficult, but are actually easier. Lines parallel to the y-axis are vertical and lines parallel to the x-axis are hortizontal. You can tell that a line is going to be parallel to an axis if it only contains *one* of the variables x or y. Horizontal lines have the form $y = c$; that is, for all values of x, $y = c$. Vertical lines have the form $x = c$; that is, for all values of y, $x = c$. Remember you may need to rearrange the equation to get it to look like this. When trying to sketch lines parallel to the axes, we need to look at whether the value c is positive or negative. This tells us which side of the axis to place the line. For example, $y = -3$ is a horizontal line through -3 on the y-axis; that means it runs parallel to and below the x-axis.



Practice Question 2

Which of the graphs below matches the equation $x - 4 = 3$?



Sometimes we are given information about an equation or we have a graph and we have to determine the equation from the information given. To determine a linear equation we need the gradient and the y-intercept. We can evaluate these if we have one of the following:

- two points on the line
- gradient and one point

Most of the time you will be able to use the gradient formula to solve for the gradient and then use substitution to find the y-intercept. However, sometimes we are only given one point and some information about the gradient. Often this occurs when we are trying to find a line that is *parallel* or *perpendicular* to a given line. If the new line is parallel to the old line, then it must have the same gradient as the old line. This makes sense: if it parallel then it is at the same slope, i.e. the lines will never cross. So, if we are looking for a new line parallel to an old line we already know the gradient of the new line: it's the same as the old line.
If a new line is to be perpendicular (at right angles) to an old line then the new line has a gradient that is the *negative inverse* of the old gradient. That is, if the old gradient was m, the new gradient is $\frac{-1}{m}$. So once again if you are looking for a line perpendicular to an existing line, you can work out the new gradient really quickly. Take the line $y = 3x + 5$. A line perpendicular to this line would have a gradient of $\frac{-1}{3}$. If we are told the new line goes through $(3, 7)$ we can substitute these values into our semi-formed equation $y = \frac{-1}{3}x + c$ and solve for c. We get $c = 8$, so our new line has the equation $y = \frac{-1}{3}x + 8$.

Practice Question 3
Find the line perpendicular to $y = -6x - 3$ that goes through the point (12,4).

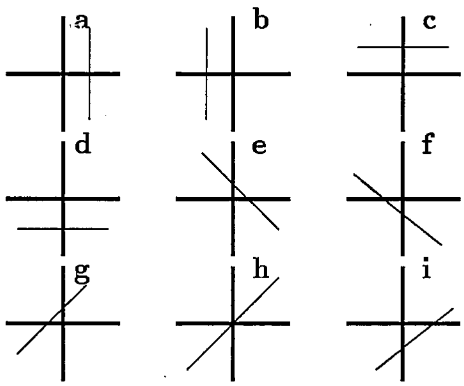
Discussion Questions
Work through these problems with the person next to you or in a small group.

Match the following equations with the graphs below.

1. $y - 2 = 4x$ 2. $y = 4$

3. $7 = y + 3x$ 4. $-3y - 6 = x$

5. $2x + 10 = 0$ 6. $2y = 8x$



Find the line that is:
7. parallel to $y = 4x + 2$ and goes through $(0, 7)$
8. parallel to $y = -3x - 2$ and goes through $(1, 2)$
9. perpendicular to $y = \frac{1}{3}x + 7$ and goes through $(2, 1)$
10. perpendicular to $y = -2x - 5$ and goes through $(3, -8)$