2.4 Solving Absolute Values

The notation $|x|$ denotes the *absolute value* of the number x. This means the **distance** between 0 and x on the real number line. Thus $|4| = 4$ and $|-3| = 3$. The formal definition of absolute value is:

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

Thus the absolute value strips away the negative sign from the whole term if present. In the case of $|x-4|$ we would have:

$$|x-4| = \begin{cases} x-4 & \text{if } x-4 \geq 0, \\ -(x-4) & \text{if } x-4 < 0. \end{cases}$$

To solve an absolute value equation we split it into two cases. Take $|x-3| = 9$; either case 1: $x-3=9$ or case 2: $x-3=-9$. The absolute value strips away the negative sign if it was there, so to solve we have to consider the case where $x-3=-9$. Once we have our two cases, $x-3=9$ or $x-3=-9$, we solve each like a normal equation and each gives an answer. For this example we get $x=12$ from case 1 and $x=-6$ from case 2.

Practice Question 1

$$|x+4| = 7$$

Discussion Questions

Work through these problems with the person next to you or in a small group.

1. Simplify $a - b + |a-b| - |b-a|$
2. Solve $|7+3a| = 11 - a$
3. A submarine is 340 metres below sea level. It has rock formations above and below it, and should not change its depth by more than 17 metres. If d is its distance below sea level, write an absolute value equation that represents this situation. Solve the equation and hence determine the safest highest and lowest depth the submarine can sit at.
4. Why is $|x+y| = 5$ not equal to $|x| + |y| = 5$?

2.7 Surds

Surds are irrational square roots, that is, they cannot be written as fractions. This can make working with them annoying unless we know how to simplify them. If you can find a factor that is a square number, eg. 4, 9, 16, you can move its square root outside the square root. Otherwise to simplify a surd we find all of its factors and try to take *pairs* of idential factors outside of the square root sign. For example, to simplify $\sqrt{84}$ we find the prime factors of 84: $84 = 2 \times 2 \times 3 \times 7$. From this we can see that the factor 2 occurs *twice*. Because we have a *pair* we can take it outside of the square root. This is because $\sqrt{2 \times 2} = \sqrt{2} \times \sqrt{2} = 2$. Therefore we have $\sqrt{84} = \sqrt{2 \times 2 \times 3 \times 7} = 2\sqrt{3 \times 7}$. There are no more pairs of factors, so we evaluate any products inside or outside the square root. So we are left with $\sqrt{84} = 2\sqrt{21}$.

Practice Question 2

Simplify

$$\sqrt{180}$$

Surds, like other numbers, can be added, subtracted, mulitplied and divided. When adding or subtracting surds remember you need to collect like terms, that is, you can only add/subtract those that have the same number under the square root sign. So $\sqrt{5} + \sqrt{7} \neq \sqrt{12}$. Instead $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$, you can think of this as 2 groups of $\sqrt{3}$ plus 5 groups of $\sqrt{3}$ equals 7 groups of $\sqrt{3}$. You may need to simplfy surds so that you have the same number under the square root sign.

Practice Question 3

Simplify

$$3\sqrt{2} + 2\sqrt{18}$$

In mulitiplication and division any numbers under a square root can be brought together under one square root. That is, $\sqrt{5} \times \sqrt{7} \times \sqrt{12} = \sqrt{5 \times 7 \times 12}$. These can then be evaluated and simplified. $\sqrt{5 \times 7 \times 12} = \sqrt{420} = 2\sqrt{105}$. The same applies to division: $\sqrt{12} \div \sqrt{3} = \sqrt{12 \div 3} = \sqrt{4} = 2$. Remember that $2\sqrt{3}$ is $2 \times \sqrt{3}$, so that $2\sqrt{3} \times 3\sqrt{4}$ is simply $2 \times 3 \times \sqrt{3} \times \sqrt{5}$. Simplifing this we get $6\sqrt{15}$.

Practice Question 4

Simplify

$$2\sqrt{2} \times \sqrt{12} \div \sqrt{3}$$

Discussion Questions

Work through these problems with the person next to you or in a small group.

1. Find the area of a box with a width of $2\sqrt{7}$ cm and a length of $9\sqrt{2}$ cm.
2. Simplify
$$\sqrt{1 + \sqrt{3 + 3\sqrt{4}}}$$
3. The square root turtle relay team has to run $\sqrt{3332}$ metres. If the first turtle completes $\sqrt{425}$ m, the second turtle runs $\sqrt{68}$ m and the third turtle runs $\sqrt{612}$ m, how far must the final turtle run to finish the race?