1.8 Fractions

Fractions can be added, subtracted, multiplied and divided. Although many calculators can perform calculations with fractions, it is important that you know how to manipulate fractions, because when a fraction involves a letter you cannot simply 'put it into your calculator'.

Addition and Subtraction of Fractions

Imagine that you had a third of a block of chocolate and your friend had a half of a block. How much chocolate do you have together?

To **add** or **subtract** fractions they must have the **same denominator**. Write each fraction as an equivalent fraction with the same denominator. Then add or subtract the numerators and place that sum over the common denominator. If possible simplify the fraction.

In our example of the block of chocolate the common denominator will be 6, so we split the block of chocolate into 6 equally-sized bits.

1/3 + 1/2 = 5/6

2/6 + 3/6 = 5/6

Practice Question

$$\frac{1}{2}+\frac{2}{3}+\frac{5}{4}$$

(Answer: $2\frac{5}{12}$)

Multiplication and Division of Fractions

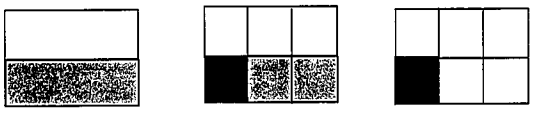
When multipling fractions, often cancelling can be done as a first step. To cancel we must find a common factor in the numerator and the denominator. For example, in the expression $\frac{1}{4} \times \frac{2}{3}$ we can start by cancelling the common factor of 2 in the 2 and the 4. So our first step would be:

$$\frac{1}{\cancel{2} \times 2} \times \frac{\cancel{2}}{3}$$

This leaves the following simplified expression which can then be evaluated.

$$\frac{1}{2} \times \frac{1}{3}$$

To evaluate this expression we need to think about multiplication of fractions. Let's think about a block of chocolate again. Say that you have half a block of chocolate and three friends. If you want to each get an equal share, then you must each get a third of the half-block. We have to find out what one third of one half of a block of chocolate is. In mathematics, 'of' often means multiply. For example, if I want three groups of four people, that is 3×4 people which is 12 people. So in this example we want a $\frac{1}{3}$ of $\frac{1}{2}$, which is $\frac{1}{3} \times \frac{1}{2}$, which is $\frac{1}{6}$.



1/2 1/3 of the 1/2 1/6

To **multiply** two fractions, first cancel if possible, then multiply the numerators to get the new numerator and multiply the denominators to get the new denominator. (There is no need for the denominators to be the same.) If possible simplify the fraction.

Practice Question

$$\frac{1}{4} \times \frac{2}{5} \times \frac{5}{6}$$

(Answer: $\frac{1}{12}$)

Division of fractions can be a little tricky. We often think of division as breaking into a certain number of groups. For example $6 \div 3$ is how many apples are in each group if I split 6 apples into 3 groups of equal size. The answer is 2 apples. Let's look at division of fractions. Think back to the block of chocolate. We had $\frac{1}{2}$ of a block and wanted to divide it among 3 friends. That is, $\frac{1}{2} \div 3$. We calculated this and the found the answer was $\frac{1}{6}$. However, the calculation we did was $\frac{1}{2} \times \frac{1}{3}$. This is the same as $\frac{1}{2} \div 3$. Multiplication and division are the opposite of each other. Therefore, one number divided by a second is the same as the first number multiplied by the *inverse* of the second number. This is how we divide fractions: we take the inverse of the **second** fraction and multiply it by the first. That is, change the divide sign to a multiply sign and flip the **second** fraction.
For example

$$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

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Practice Question

$$\frac{1}{4} \div \frac{2}{5} \div \frac{5}{6}$$

(Answer: $\frac{3}{4}$)

Discussion Questions
Work through these problems with the person next to you or in a small group.

1. Simplify

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1+1}}}$$

2. How many times do you have to write $\frac{1}{3}$ to make the following statement true?

$$\frac{1}{3} \div \frac{1}{3} \div \frac{1}{3} \div \cdots \div \frac{1}{3} = 81$$

3. Sarah was hungry one night and looked into the freezer to find a four-litre tub of ice cream. She started eating the ice cream, but so she would not get in to trouble for eating it all, she stopped when she had eaten half of it. The next night she was hungry again, so she went back to the ice cream and again stopped after eating half of what was there. This went on for five nights in a row before her mother caught her. How much ice cream was left at the end of the five nights?

4. Alex decides to build his pet pig a pen. He needs 20 metres of fencing in total. On Saturday morning he goes to the hardware store and buys six posts each $\frac{1}{6}$metre wide, four red railings each $\frac{3}{4}$ metre long, twelve blue railings each $\frac{5}{8}$ metre long and a green gate 2.5 metres wide.
How many black railings of $\frac{1}{2}$ metre long does he need to buy to finish his pig pen?

Answers

1. $1\frac{3}{5}$ 2. 6 3. $\frac{1}{32}$ of 4L 4. 12