A distributed approach to bandwidth allocation in logical networks

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Outline

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2. Problem framework
3. Buy/Sell heuristic
4. Continuous re-allocation scheme
5. Discretised re-allocation scheme
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Motivation

• Nodes $A \rightarrow F$
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- Connected by a set $L$ of physical links, each with capacity $c_l$
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- Connected by a set $L$ of physical links, each with capacity $c_l$
- A route $r$ is a non-empty subset of the physical links, with Poisson arrival rate $\lambda_r$ and mean connection time $\mu_r^{-1}$
Motivation

- Routes require capacity to carry traffic
- There are situations where one physical link is used in many paths through the network
- However, the physical links have capacity constraints
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The natural question that arises is

How should capacity be allocated in a network between competing streams of traffic?
Problem framework

- A physical network overlaid by a logically sufficiently connected network
- When traffic arrives, we do not “check ahead” for spare capacity
- Reserve capacity between each origin and destination node ($A \rightarrow F$)
- This allows us to decouple the network and treat each route $r$ as an Erlang loss system with capacity $x_r$
  - A continuous time Markov chain, where arrivals that find the system at capacity are lost; they are not queued
Allocating capacity across routes in the network requires respect of capacity contraints. Define a matrix $A$ with elements

$$A_{lr} = \begin{cases} 1 & \text{if } l \text{ is in } r \\ 0 & \text{otherwise} \end{cases}$$

Assume each route $r$ has an associated utility function $U_r(x_r)$ and we wish to maximise utility over the network; ex. optimise quality of service over all routes.
Problem framework

The optimisation formulation for the network as a whole is:

\[
\max_{x_r} \sum_{r \in \mathcal{R}} U_r(x_r)
\]

subject to

\[
Ax = C
\]

\[
x \geq 0.
\]

Ex. \( U_r(x_r) = \lambda_r \theta_r T (1 - E(\rho_r, x_r)) \) and \( E(\rho_r, x_r) \) gives the blocking probability on route \( r \).
Kelly’s approach

*Kelly et al (1998) Rate control for communication networks: shadow prices, proportional fairness and stability*

- Different physical context, but similar mathematical formulation (key difference: inequality constraint)
- Mathematically tractable problem, but there exist centralisation issues
- Decomposition into a USER and NETWORK problem, that are tied together using Lagrangian arguments
- Using these ideas, a rate control algorithm is constructed
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Can we use a similar approach, but exploit our network structure in the solution method?
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Buy/Sell heuristic

- Using the utility function, we can derive buy and sell prices of a unit of capacity
- Assuming a route has capacity $x$,

\[
\text{BUY}(x) = U(x + 1) - U(x) \\
\text{SELL}(x) = U(x) - U(x - 1).
\]

Ex. The buy price using the previous example would take the form

\[
\text{BUY}(x) = \theta \lambda T (E(\rho, x) - E(\rho, x + 1))
\]
Buy/Sell heuristic

- *Direct routes* are those with origin and destination nodes connected by one physical link only.

The proposed capacity trading scheme operates locally, that is transit routes can only trade with their constituent direct routes.
Buy/Sell heuristic

- **Direct routes** are those with origin and destination nodes connected by one physical link only.
- **Transit routes** are those which utilise more than one physical link only.
Buy/Sell heuristic

- **Direct routes** are those with origin and destination nodes connected by one physical link only.
- **Transit routes** are those which utilise more than one physical link.
- The proposed capacity trading scheme operates *locally*, that is transit routes can only trade with their constituent direct routes.
Buy/Sell heuristic

- Chiera and Taylor (2002) derived a capacity value function to be used for this type of trading scheme
  - Modelled each route as an $M/M/C/C$ queue
- Chiera et al (2003) showed that this type of local interaction (between transit and direct routes) lowers blocking probabilities of a network
- At this stage, the method is a heuristic – it is not clear whether the global optimum can be reached
- Can we provide some theoretical support to this heuristic?
1. Motivation
2. Problem framework
3. Buy/Sell heuristic
4. **Continuous re-allocation scheme**
5. Discretised re-allocation scheme
6. Work in progress
Global problem (revisited)

We can split the set of routes \( R \) into \( T \), the set of transit routes and \( D \), the set of direct routes.

\[
\max_x \left\{ \sum_{r \in T} U_r(x_r) + \sum_{l \in D} U_l(x_l) \right\}
\]

subject to

\[
\sum_{r \in T \cup D : l \in r} x_r = C_l \quad \forall l \in L
\]

\[
x \geq 0.
\]
Reformulation

We can rewrite the global problem solely in terms of capacity on the transit routes.

\[
\max_x \left\{ \sum_{r \in T} U_r(x_r) + \sum_{l \in D} U_l \left( C_l - \sum_{r:l \in r} x_r \right) \right\}
\]

subject to

\[
\sum_{r \in T : l \in r} x_r \leq C_l \quad \forall l \in L
\]

\[
x \geq 0.
\]
Karush-Kuhn-Tucker conditions

A Karush-Kuhn-Tucker point in this case will be the global optimum on the feasible region, with capacities $x$ and Lagrange multipliers $\lambda$ and $\eta$ satisfying

1. $U'_r(x_r) - \sum_{l:l \in r} U'_l \left( C_l - \sum_{r:l \in r} x_r \right) - \sum_{l:l \in r} \lambda_l + \eta_r = 0$

2. $\lambda_l \geq 0$ and $\eta_r \geq 0$

3. $\lambda_l \left( C_l - \sum_{r:l \in r} x_r \right) = 0$ and $\eta_r x_r = 0$ (C-S)
Karush-Kuhn-Tucker conditions

- Assume first that $x_l > 0$ and $x_r > 0$ for all routes in the network.
- The KKT conditions specify, the optimal allocation satisfies:
  \[ U'_r(x_r) = \sum_{l: l \in r} U'_l \left( C_l - \sum_{r: l \in r} x_r \right) \]
- This is equivalent to a trading scheme where “infinitesimal” chunks of capacity can be traded.
- Encouragement that the buy/sell heuristic was on the right track.
Dynamics of capacity

Let the dynamics of transit route capacity be described by the system below

\[
\frac{dx_r}{dt} = \kappa \left( U_r'(x_r) - \sum_{l \in r} U_l'(C_l - \sum_{s:l \in s} x_s) - \sum_{l \in r} \lambda_l + \eta_r \right)
\]

- The fixed point of this system is equivalent to the KKT point
Dynamics of capacity

Let the dynamics of transit route capacity be described by the system below

\[ \frac{dx_r}{dt} = \kappa \left( U'_r(x_r) - \sum_{l \in r} U'_l(C_l - \sum_{s:l \in s} x_s) - \sum_{l \in r} \lambda_l + \eta_r \right) \]

- The fixed point of this system is equivalent to the KKT point
- What are the parameters \( \lambda_l \) and \( \eta_r \)?
Continuous scheme

• Applying the $l_2$ penalty method to our optimisation problem helps solve this problem

\[
\lim_{k \to \infty} k \left( \sum_{s:l \in s} x_s - C_l \right)^+ = \lambda_l^* \\
\lim_{k \to \infty} k (-x_r)^+ = \eta_r^*
\]

• From a practical perspective, we cannot set $k \to \infty$. Instead, we choose a large value $K$
Continuous scheme

- The fixed point of the system is *exactly* the optimal solution, when the solution is in the strict interior of the feasible region.
- When the solution is on the boundary, setting $K$ to be very large, arbitrarily closely approximates the optimal solution.
Continuous scheme

- The fixed point of the system is \textit{exactly} the optimal solution, when the solution is in the strict interior of the feasible region
- When the solution is on the boundary, setting $K$ to be very large, arbitrarily closely approximates the optimal solution
- The fixed point is \textit{attracting} – this is established using Lyapunov arguments
We consider a network with

- 4 direct routes

Each route has a utility function dependent on parameters $r$, the arrival rate and $r'$, the revenue generated from each accepted arrival, and is a function of capacity.
Network example

We consider a network with

- 4 direct routes
- 6 transit routes; it is logically fully connected
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- 6 transit routes; it is logically fully connected
- Each route has a utility function dependent on parameters $\lambda_r$, the arrival rate and $\theta_r$, the revenue generated from each accepted arrival, and is a function of capacity
Network example

The results using the continuous trading scheme are shown below.

![Graph of dynamics of transit route capacity and capacity exhausted by transit routes over time.](image-url)
If the optimal solution is a boundary solution, the results ($K = 100$) are shown below.
1. Motivation
2. Problem framework
3. Kelly’s capacity allocation method
4. Buy/Sell heuristic
5. Continuous re-allocation scheme
6. Discretised re-allocation scheme
7. Work in progress
Discretised system

- Assume that trades can only occur in amounts of \( \Delta \) units at a time
- As described in the buy/sell heuristic, a trade occurs if one of the following conditions is satisfied:
Discretised system

- Assume that trades can only occur in amounts of $\Delta$ units at a time
- As described in the buy/sell heuristic, a trade occurs if one of the following conditions is satisfied:

\[
Buy(x_r) > \sum_{l \in r} Sell(C_l - \sum_{s:l \in s} x_s)
\]

\[
\Rightarrow x_r := x_r + \Delta
\]
Discretised system

- Assume that trades can only occur in amounts of $\Delta$ units at a time
- As described in the buy/sell heuristic, a trade occurs if one of the following conditions is satisfied:

$$
\sum_{l \in r} Buy(C_l - \sum_{s : l \in s} x_s) > Sell(x_r)
$$

$$
\Rightarrow x_r := x_r - \Delta
$$
Discretised system

- Assume that trades can only occur in amounts of $\Delta$ units at a time.
- As described in the buy/sell heuristic, a trade occurs if one of the following conditions is satisfied:
- If neither condition is satisfied, no trade occurs.
Discretised system

- If a route has close to zero (or $< \Delta$) allocated capacity and the conditions state the route must relinquish capacity, the trade occurs.
- However this yields an allocation that is not feasible.
Discretised system

- If a route has close to zero (or $< \Delta$) allocated capacity and the conditions state the route must relinquish capacity, the trade occurs.
- However this yields an allocation that is not feasible.
- The next time prices are calculated, they will involve a penalty term, taking the same form as in the dynamical system.
- The relinquishing route will be able to acquire capacity at the next trade.
Network example

Allowing transit routes to instigate trading in a random order,
Network example

However, the system can also evolve to
Oscillatory behaviour arises when a solution is on the boundary of the feasible region
Network example

Introduce a threshold $\epsilon$ by which buy prices have to exceed sell prices before a trade occurs.
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Future work

• Can $\Delta$ be chosen so that the number of stable equilibria in the discrete system reduces to one? Analyse as the number of absorbing states in a finite-state Markov chain.

• How “far” from the optimal can we say an equilibrium solution is, given multiple stable equilibria?

• How can $\epsilon$ be chosen using local information, and how much does it affect optimality?
Future work

- Can $\Delta$ be chosen so that the number of stable equilibria in the discrete system reduces to one? Analyse as the number of absorbing states in a finite-state Markov chain.
- How “far” from the optimal can we say an equilibrium solution is, given multiple stable equilibria?
- How can $\epsilon$ be chosen using local information, and how much does it affect optimality?
- Model situation where occupancy on each route $r$ is fluctuating stochastically through modified utility function.
- Performance analysis of schemes.