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What is a mainland-island metapopulation?

A mainland-island metapopulation is a group of populations that occupy geographically separate habitat patches (*islands*) which are colonised by migrants from an outside source population (*mainland*).

Why model these systems?

We model these systems to study their dynamic behaviour and determine conditions which explain metapopulation persistence or extinction. This information helps us to estimate the viability of the metapopulation and to devise strategies to improve its survival.

Parameters

- N total number of patches in the metapopulation
- t discrete time $(0, 1, 2, 3, \dots)$
- e probability that an occupied patch goes extinct $(0 < e < 1)$
- c probability that an empty patch is colonised $(0 < c < 1)$

The Markov Chain

Number of occupied patches at time t :

$$n_t^{(N)} \in \{0, 1, 2, \dots, N\}$$

Proportion ($= n_t^{(N)}/N$) of patches occupied at time t :

$$X_t^{(N)} \in \{0, 1/N, 2/N, \dots, 1\}$$

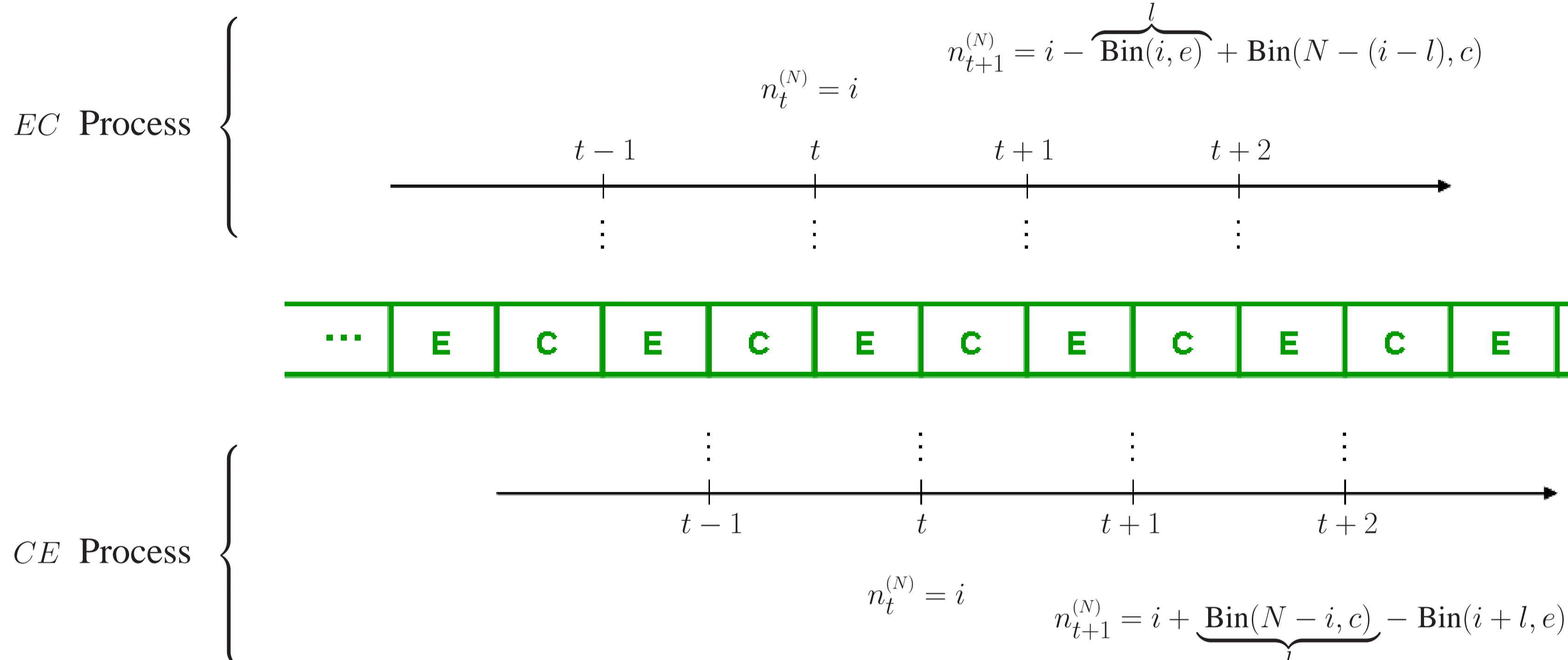
The Model

We separate the extinction and colonisation events to occur in two separate phases. The extinction phase is governed by the transition matrix E and the colonisation phase is governed by the transition matrix C .

If the census is taken after the C phase, then $P = EC$.

If the census is taken after the E phase, then $P = CE$.

The number of events that occur in the second phase is dependent on the number of events that occur in the first phase.



Distribution of $n_t^{(N)}$

Parameter	EC Process	CE Process
p	$1 - e(1 - c)$	c
q	$1 - e$	$c(1 - e)$

Given $n_0^{(N)} = i$, and letting $a = p - q = (1 - e)(1 - c)$, then

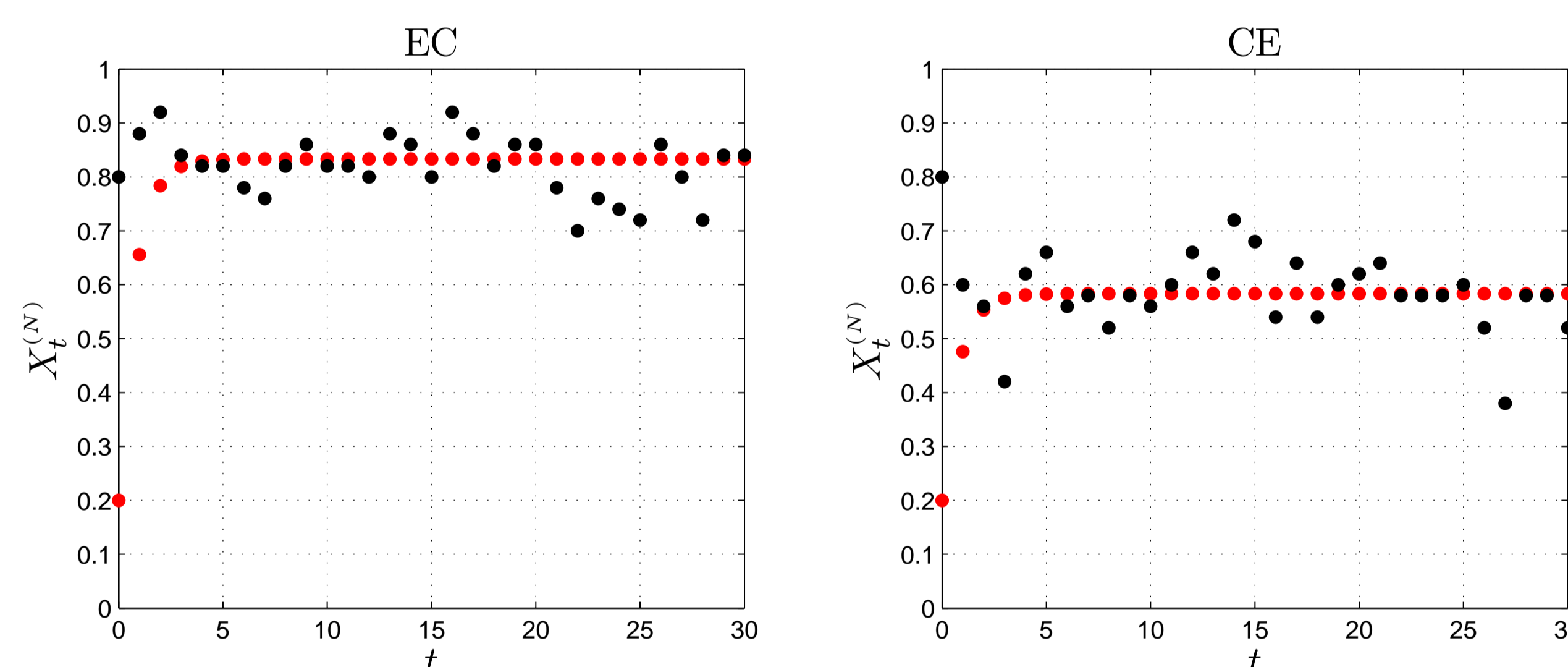
$$n_t^{(N)} \stackrel{D}{=} \text{Bin}(i, p_t) + \text{Bin}(N - i, q_t)$$

$$\text{where } p_t = q_t + a^t \text{ and } q_t = \frac{q(1 - a^t)}{1 - a}$$

For the simulations shown here we have let $e = 0.3$, $c = 0.6$, $x_0 = 0.2$, $X_0^{(N)} = 0.8$ and $N = 50$.

Law of Large Numbers

If $X_0^{(N)} \rightarrow x_0$ as $N \rightarrow \infty$, then $(X_t^{(N)}) \Rightarrow (\mathbf{x}_t)$ as $N \rightarrow \infty$, where $\mathbf{x}_t = a^t \mathbf{x}_0 + \mathbf{q}_t$.

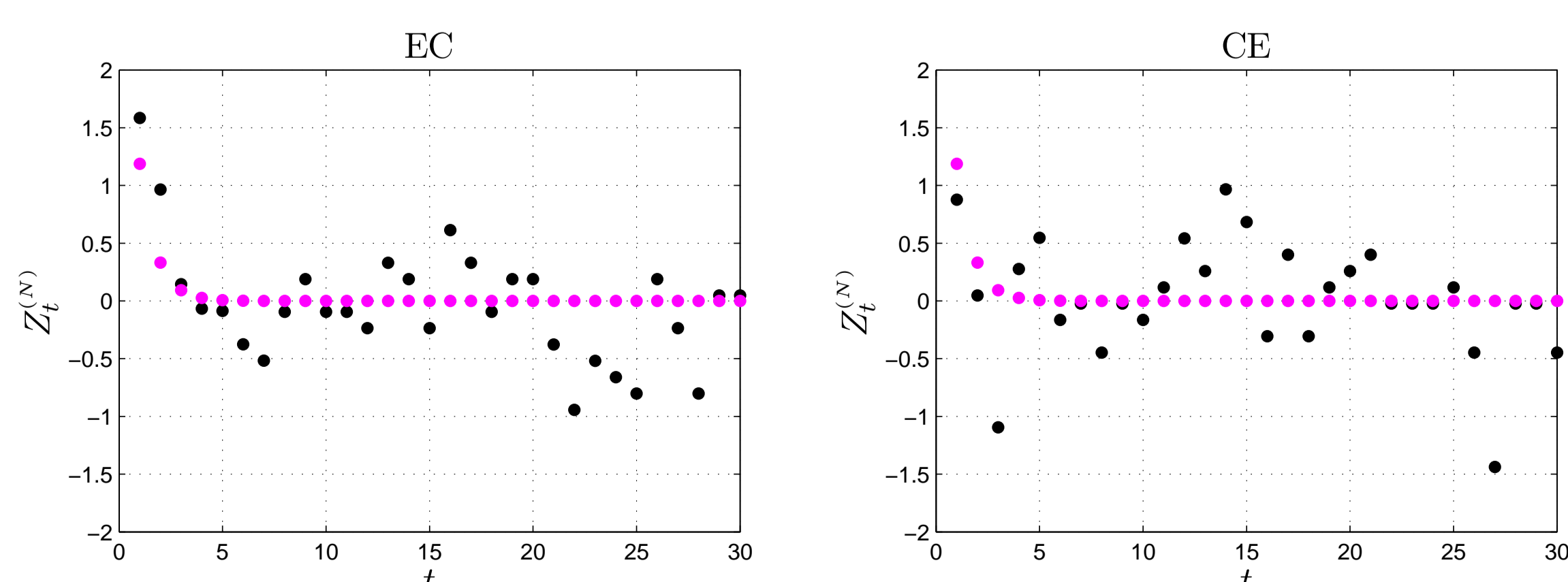


Central Limit Theorem

Define $Z_t^{(N)} := \sqrt{N}(X_t^{(N)} - \mathbf{x}_t)$. If $\sqrt{N}(X_0^{(N)} - x_0) \rightarrow z_0$ as $N \rightarrow \infty$ then

$$(Z_t^{(N)}) \Rightarrow (Z_t) \text{ as } N \rightarrow \infty$$

where (Z_t) is the AR(1) process given by $Z_{t+1} = aZ_t + \text{Normal}(0, \sigma_t^2)$ with $\sigma_t^2 = p(1 - p)\mathbf{x}_t + q(1 - q)(1 - \mathbf{x}_t)$.
Therefore $Z_t \sim \text{Normal}(a^t z_0, v_t)$ where $v_t = p_t(1 - p_t)x_0 + q_t(1 - q_t)(1 - x_0)$.



Conclusions and Future Work

- Given the initial number of occupied patches, the number at time t is the sum of two *independent* binomial random variables.
- For large N , the proportion of occupied patches at time t is asymptotically deterministic.
- The fluctuations around the deterministic trajectory, scaled by \sqrt{N} , are approximately normally distributed for large N .
- These results apply to both the *EC* and *CE* cases. The only difference between the two cases are the parameters p and q .
- Next we will use the AR(1) process to evaluate MLEs for the parameters e and c .