Assignment 3

Insert author

Insert date

Abstract

Instructions: Please set the following. Make sure to load the amsmath package. Experiment with the following commands:

```
\usepackage{times}  \Omega  \mathcal{F}  \omega  \leq  \{ \}
\Pr  {\bf bold text}
```


If F is a random variable on a probability space $(\Omega, \mathcal{F}, P)$, then

$$\begin{aligned}\Pr(X \leq x) :&= P(\{X \leq x\}) \\ &= P(\{\omega \in \Omega : X(\omega) \leq x\}).\end{aligned}$$

For example,

$$\Pr(X \leq x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{x}{2\pi}, & \text{if } 0 \leq x < 2\pi, \\ 1, & \text{if } x \geq 2\pi. \end{cases}$$

The two state Markov chain. Let $S = \{0, 1\}$ and let

$$P = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix},$$

where $p, q \in (0, 1)$. After diagonalizing P, we see that

$$\begin{aligned}P^{(n)} &= \left[\frac{1}{p+q}\begin{pmatrix} 1 & p \\ 1 & -q \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix}\begin{pmatrix} q & p \\ 1 & -1 \end{pmatrix}\right]^n \\ &= \frac{1}{p+q}\begin{pmatrix} 1 & p \\ 1 & -q \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & r^n \end{pmatrix}\begin{pmatrix} q & p \\ 1 & -1 \end{pmatrix} \\ &= \frac{1}{p+q}\begin{pmatrix} q+pr^n & p-pr^n \\ q-qr^n & p+qr^n \end{pmatrix},\end{aligned}$$

where $r = 1 - p - q$. Thus, we have an *explicit expression* for the n-step transition probabilities.