A polynomial

The polynomial P we have choosen in this example is nothing special but it suits our purposes because it changes fairly rapidly on $[0,\ 8]$.

$$P(x) = 8.7283509.10^{-3}x^7 - 0.2626950683x^6 + 3.147340034x^5 - 18.98393186x^4 + 59.68279449x^3 - 89.75801494x^2 + 47.66577899x + 1 \quad (1)$$

In order to choose a suitable gauge we estimate

$$\left| \int_u^v P - P(x)(v-u) \right| \leq |P(\xi) - P(x)|\,(v-u),$$

with some $\xi \in [u,\ v]$. Now we use the Taylor formula in which we neglect all terms of order 3 and higher and have:

$$|P(\xi) - P(x)| \leq \left| P'(x)(\xi - x) + \frac{P''(x)}{2}(\xi - x)^2 \right|.$$

If a tagged division $\{[u,v],\ x\}$ is δ-fine (for some not yet specified δ) then $|\xi - x| \leq \delta(x)$. We now decide (a bit arbitrarily but quite reasonably) that we shall have $\delta \leq 0.6$. Then

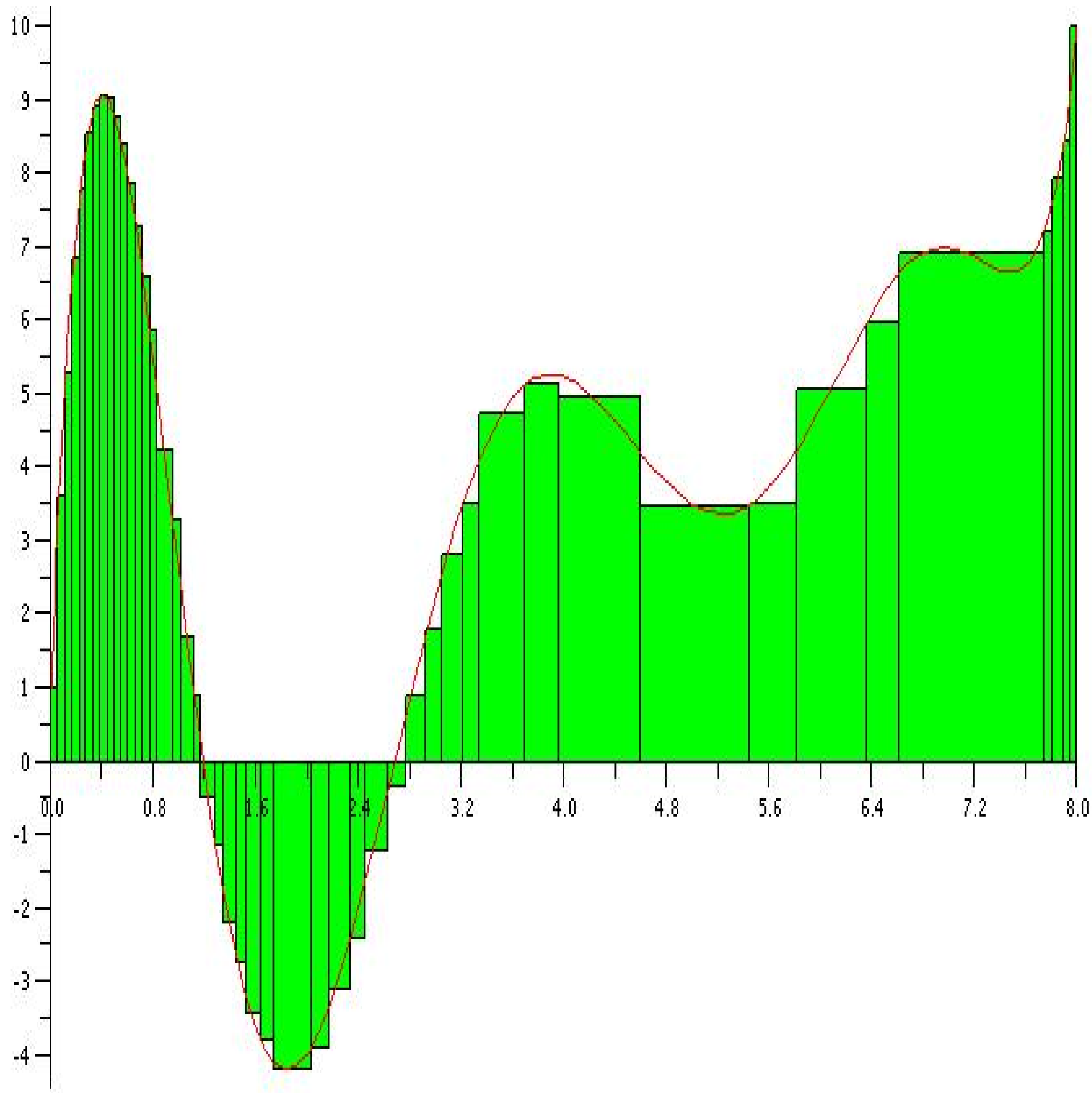
$$|P(\xi) - P(x)| \leq \mathrm{Min}\,(0.6,\ (|P'(x)| + 0.3\,|P''(x)|))\,\delta(x).$$

For an arbitrary $\varepsilon > 0$ the choice for δ as

$$\delta(x) = \mathrm{Min}\left(0.6,\ \frac{\varepsilon}{|P'(x)| + 0.3\,|P''(x)|}\right)$$

seems quite natural. Unfortunately, even with $\varepsilon = 1$ some of the subintervals of the δ-fine tagged division become too small for the display. To correct this we made the length of the smallest subinterval at least half a mm. Finally we have

$$\delta(x) = \mathrm{Max}\left(0.05, \mathrm{Min}\left(0.6,\ \frac{1}{|P'(x)| + 0.3\,|P''(x)|}\right)\right).$$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