SCIE1000, Tutorial Week 5, Semester 1, 2011.

Aims

- This week you will work through some general calculation and discussion questions.
- As usual, you should recognise that the broad concepts and techniques we cover are more important than the specific examples. Do not try to commit lots of facts to memory; instead, know **how** to do things, and **when** certain models and approaches are appropriate.
- The initial submission of your project is due next week. You really should be making excellent progress by now. **The requirements for submission, including the due date and time, are very strict.** Do not leave it until the last minute!

1 Questions

1. (Final exam, 2008. Worth 8 marks, so about 8 minutes to work.) A young child suffered an asthma attack and was hospitalised. The doctor used a spirometer to monitor her respiration. Her tidal volume was 0.4 L, her residual capacity 1.6 L, and a complete inhalation-exhalation cycle took 4 s.

   (a) Draw a rough sketch modelling her lung capacity, $V(t)$, as a function of time. (Put labels and scales on your axes, and show two full inhalation-exhalation cycles.)

   (b) Write an equation for $V(t)$.

   (c) After ventolin treatment, her respiration rate slowed to 10 breaths per minute and her tidal volume increased to 0.6 L. Write a new equation for $V(t)$.

   (d) Briefly describe the meaning of the changes in the values of the parameters in the two equations in Parts (b) and (c).

2. The planet Neptune has a similar tilt to that of Earth, so has a similar seasonal pattern (but with seasons of different durations, of course). It takes about 16 earth hours for Neptune to rotate around its axis, and about 165 earth years to revolve around the sun.

   Derive an equation for the number of earth hours of "daytime" on Neptune on each day of a Neptune year. You may assume that the Neptune year starts on any Neptune day that you like.

3. (Final exam, 2008. Worth 4 marks, so about 4 minutes to work.) Two species of algae are growing exponentially. Species A has an initial population of 1000 per mL, and Species B has an initial population of 3000 per mL. The growth rate of Species A is 3% per hour, and the growth rate of Species B is 1% per hour. At what time will the population sizes be equal?

4. (Final exam, 2010. Worth 11 marks, so about 11 minutes to work.) A species of bacteria reproduces by individuals splitting into two. A population of this bacteria has a constant doubling time. At time $t = 0$ hours the population is 10^4 individuals, and at time $t = 5$ hours the population is 10^5 individuals.

   (a) Find the average rate of change of the population size between $t = 0$ and $t = 5$.

   (b) How many generations are there between $t = 0$ and $t = 5$? (Give your answer correct to one decimal place.)

   (c) Find the doubling time of this population.

   (d) At what time will there be 10^8 individuals in the population?

5. (Final exam, 2009. Worth 8 marks, so about 8 minutes to work.) Recall that the equation for a species area curve is $S(a) = Ca^p$, where $S(a)$ is the number of species observed in an area of size a, and C and p are constants. An ecologist conducts a species-area survey of a habitat, and plots a graph with $\log_{10}$ of the observed number of species on the y axis and $\log_{10}$ of the area on the x axis.

   (a) The ecologist notices that the graph is linear. Show mathematically why this is, and interpret the physical meaning of the y-intercept and the slope of the graph.

   (b) Two of the points on the above graph are $(0, 1)$ and $(2, 2)$. Find the values of C and p in the equation $S(a) = Ca^p$ for this habitat.

6. The Widmark formula for blood alcohol concentration (BAC) is

$$B(t) = \frac{A}{rW} \times 100 - 0.015t$$

where $B(t)$ is the BAC % of a person of weight W in g, t hours after consuming A grams of alcohol, and r gives the percentage of body weight that is water. In this formula, the absorption term assumes that the body absorbs alcohol **immediately** after consumption. The following variation, which we cover later in lectures, takes into account absorption time:

$$B = \frac{A}{rW} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$$

where k is the rate at which the body absorbs alcohol. Typically, if alcohol is consumed on an empty stomach, then $k \approx 6$ hr^{-1}. If the stomach contains food, $k \approx 2.3$ hr^{-1}.

A female who weighs 38.5 kg and is 1.55 m tall, very rapidly consumes 2 standard drinks. Estimate her BAC 10 minutes after consuming the alcohol, for the case where her stomach contains food, and also when it does not contain food. Find the difference between the two BACs. (Hint: for females, the Widmark factor r is estimated by $r = 0.31223 - 0.006446W + 0.4466H$. Also, note that 10 minutes is $1/6$ of an hour.)

7. The World Fact Book [www.cia.gov/library/publications/the-world-factbook] gives the following demographic information for Australia (around 2008–9): current population = 21,262,641; nett migration rate = 6.23 individuals per 1000; birth rate = 12.55 per 1000; death rate = 6.68 per 1000. Using these figures:

   (a) What is the current annual percentage growth rate of the Australian population?

   (b) Assuming the population is growing exponentially, and the growth rate remains constant, write an expression for Australia's population t years from now.

   (c) How long does the model predict it will take for Australia's population to exceed 25 million for the first time?

   (d) For Japan, the corresponding figures are: population = 127,078,679; nett migration is nil; birth rate = 7.87 per 1000; death rate = 9.26 per 1000. The figures show that Japan has a substantially higher death rate than Australia, but people living in Japan have a reputation for being healthier than people in many other countries, and certainly have a higher life expectancy than Australians. Explain briefly why these two facts are not inconsistent.

   (e) If Australia's and Japan's growth rates both remain unchanged over time (which is very unlikely), at what time will Japan's population equal four times that of Australia (round your time to three decimal places).

The end