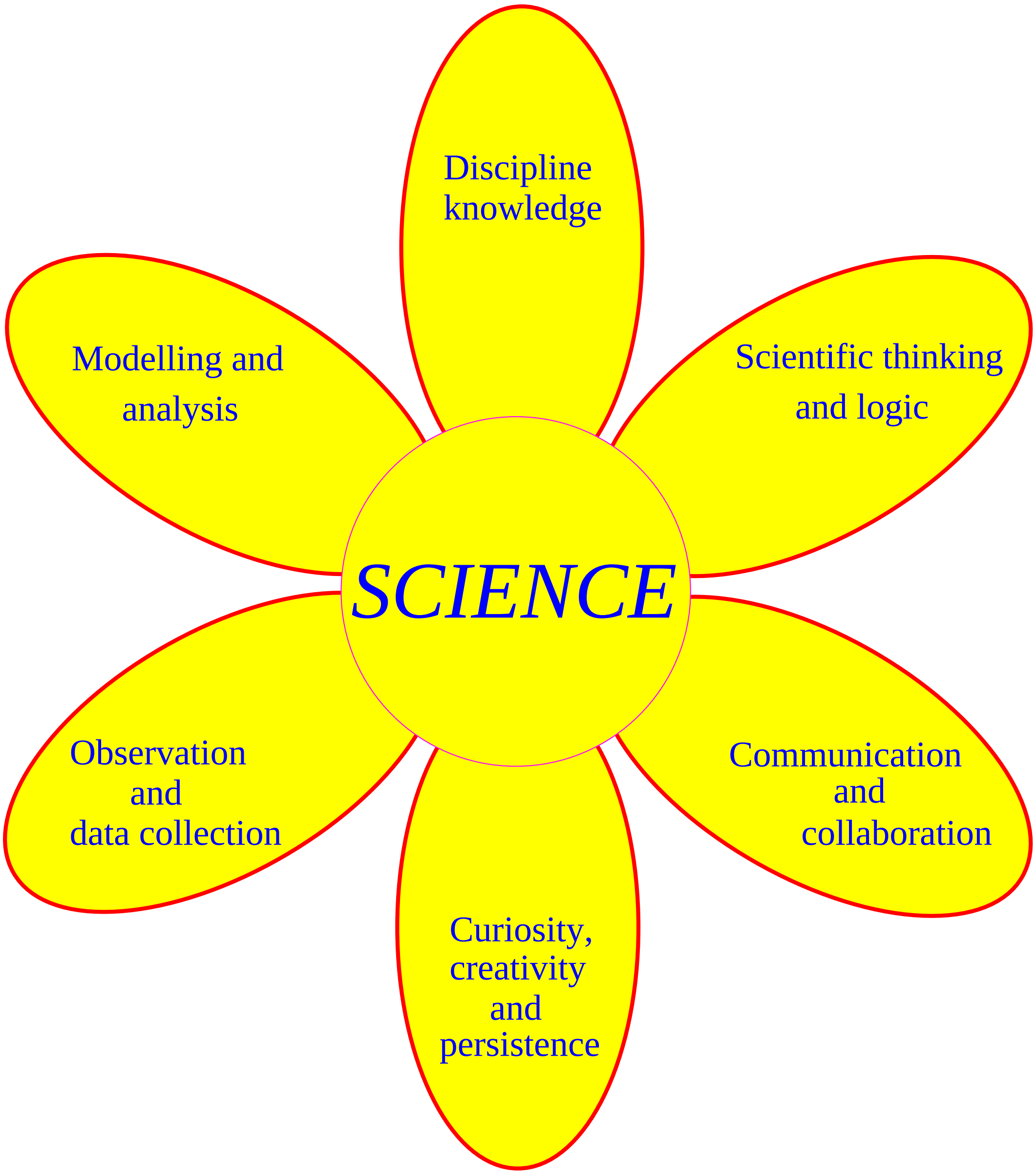
SCIE1000

Theory and Practice in Science



Course materials

Fifth edition, 2012

About these notes

This book contains the lecture notes and Python programming manual for SCIE1000. We will use these notes extensively, so it is essential that you have your own copy. Details on how you can obtain a copy will be given in class during the first week of semester. Please note that there is no text book for SCIE1000, so these notes are your primary source of information. Do not try to re-use a copy from your friends or from a previous semester: the notes change from year to year, and it is very important for you to write things in your own words.

If you lose these notes then you might have big problems. Write your name and some contact details on the bottom of this page so they can be returned to you.

These notes have been prepared very carefully, but there will inevitably be some (hopefully minor) errors in them. We are continually trying to improve the notes; if you have any suggestions, please tell us.

These notes are important. If you find them, please return them to me! My contact details are:

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Part 1: Understanding

“Before me there were no created things,
Only eterne, and I eternal last.
All hope abandon, ye who enter in!”

These words in sombre colour I beheld
Written upon the summit of a gate;
Whence I: “Their sense is, Master, hard to me!”

And he to me, as one experienced:
“Here all suspicion needs must be abandoned,
All cowardice must needs be here extinct.

We to the place have come, where I have told thee
Thou shalt behold the people dolorous
Who have foregone the good of intellect.”

And after he had laid his hand on mine
With joyful mien, whence I was comforted,
He led me in among the secret things.

Divine Comedy (1308 – 1321), Dante Alighieri (c.1265 – 1321).
(Translation: Henry Wadsworth Longfellow.)

Image 0.1: *The Hands of God and Adam* (1508 – 1512), Michelangelo (1475 – 1564), Sistine Chapel ceiling, Apostolic Palace, Vatican. (Source: en.wikipedia.org)

Chapter 1: A short discussion of nearly everything

Gaudeamus igitur, Iuvenes dum sumus
Post iucundam iuventutem, Post molestam senectutem
Nos habebit humus, Nos habebit humus.
Vivat Academia, Vivant professores
Vivat membrum quodlibet, Vivant membra quaelibet
Semper sint in flore, Semper sint in flore.

Artist: traditional (www.youtube.com/watch?v=aLUKfU2AOBY)

Image 1.1: *The School of Athens* (1510 – 1511), Raphael (1483 – 1520), Stanze di Raffaello, Apostolic Palace, Vatican. (Source: en.wikipedia.org.)

* To emphasise that science and knowledge play fundamental roles in human history, culture and society, the notes include scientifically relevant cultural experiences. *The School of Athens* depicts some famous scientists, mathematicians and philosophers, including Plato, Aristotle, Euclid, Socrates and Pythagoras.

1.1 Course rationale

SCIE1000 covers a wide range of topics. At first you might not see how all of these tie together, but the relationships are surprisingly close. The key areas covered include:

- specific problems and issues in a range of science disciplines;
- how to design, formulate and test models;
- mathematical techniques;
- computer programming;
- quantitative reasoning and critical evaluation; and
- the nature of science and scientific thinking.

Rather than requiring memorisation of specific facts, the goal of SCIE1000 is to help you learn various conceptual, scientific, mathematical and computational techniques, and how these can be applied to a wide range of disciplines.

It is likely that you will find some concepts harder than other concepts, and some areas will be of more immediate interest to you than others. Due to time constraints it is not possible to illustrate every concept with an example from each field of science. Instead, the course is divided into five broad topics: the nature of science and scientific modelling; climate and climate change; scientific thinking; drugs; and life, death and populations. There are also numerous examples from other areas of science: the techniques covered in SCIE1000 are important in *all* areas of science!

Almost every example and case study is either taken from a research paper, or is based on actual experiments, or is a fairly accurate model of a real situation. Unlike many courses, examples are generally not contrived or "made up". For example, when a quadratic equation is used to give a very good model of the probability of dying from breast cancer (if you are female), the equation genuinely models *real* data. You can estimate **your** probability by substituting your age into the equation.

1.2 Us

Professor Peter Adams is Associate Dean (Academic) in the Faculty of Science. When he is not busy with administrative things he is a mathematician in the School of Mathematics and Physics. He studied mathematics, computer science and commerce at The University of Queensland, and completed a PhD in mathematics at UQ in 1995. His area of research specialisation is combinatorial mathematics and computing. Combinatorial mathematics is concerned with selecting and arranging objects subject to constraints; problems involving this kind of activity arise in a range of practical applications. Thus his research work spans pure mathematics, computational algorithms and bioinformatics. Some of his recent research projects include using combinatorial methods for identifying drug lead molecules, and statistical methods for genome analysis. He has published over 90 scientific research papers, is an Associate Fellow of the Australian Learning and Teaching Council, and is Secretary of *Science and Technology Australia*.

(−17 °C + tongue + metal pole = idiot)

Photo 1.1: Left: tongue on pole, Finland. Right: Yellowstone Park, USA. (Source: PA.)

Professor Peter O'Donoghue (POD) is a parasitologist in the School of Chemistry and Molecular Biosciences in the Faculty of Science. He trained in cell biology at the University of Adelaide, medical parasitology at the University of Munich and veterinary parasitology at the Hannover Veterinary University. He worked at the Institute of Medical and Veterinary Science in Adelaide before moving to UQ in 1994. His area of specialisation is clinical protozoology and he practices as a diagnostician; identifying protozoan parasites causing disease in vertebrate hosts. His goal is to characterise those species occurring in Australia, the last great unexplored bastion for micro-fauna. He conducts research on the morphology, biology, phylogeny and pathogenicity of protozoan species; including sporozoa, ciliates, flagellates and amoebae in the blood, gut and tissues of mammals, birds, reptiles and fish. He uses conventional and modern technologies to study organismal, cellular and molecular biology, including light and electron microscopy, immunoassays, biochemical profiles and nucleotide analyses. He has published over 150 scientific papers in five main areas of research: cyst-forming sporozoa in domestic animals; enteric coccidia and haemoprotozoa in wildlife; protozoa affecting aquaculture; endosymbiotic ciliates in herbivores; and protozoal biodiversity. He was awarded a Doctor of Science by UQ and was elected Fellow of the Australian Society for Parasitology.

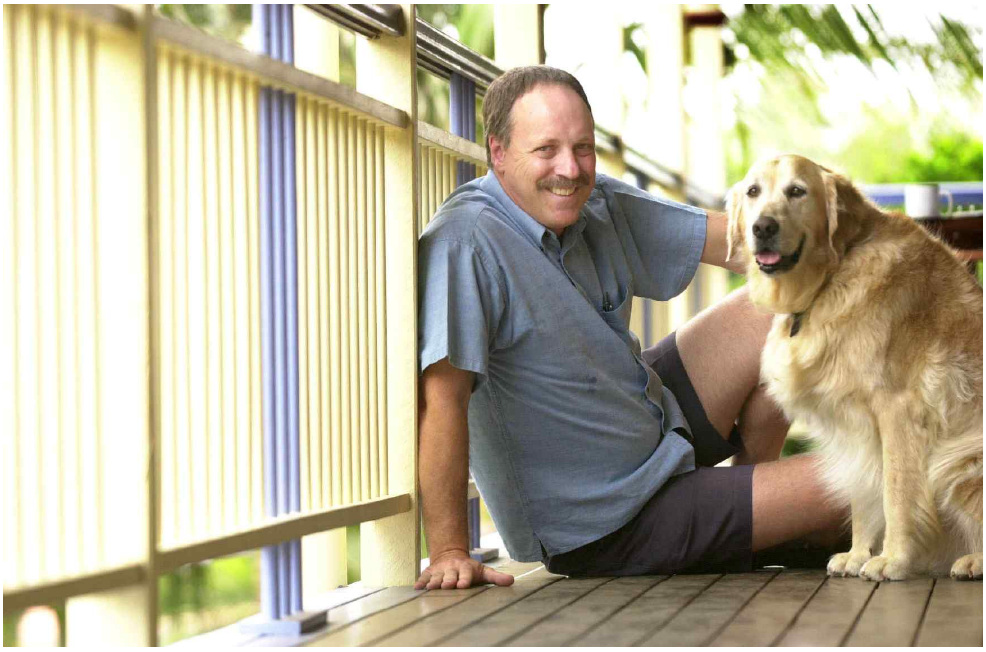


Photo 1.2: Cute picture of dog (and POD). (Source: POD.)

Associate Professor Phil Dowe is a Reader in Philosophy in the Faculty of Arts. He studied Physics, History and Philosophy of Science for a BSc at the University of New South Wales, and has a PhD in Philosophy from Sydney University.

He teaches *Introduction to Philosophy*, *Time Travel*, *Chance Coincidence and Chaos*, *Science and Religion*, *Philosophy of the Life Sciences* and *Advanced Philosophy of Science*. His main areas of research are philosophy of science and metaphysics. His books include *Physical Causation* (Cambridge 2000) and *Galileo, Darwin, Hawking* (Edinburgh, 2005). He has published papers on causation, chance and time.

When pushed to divulge something interesting about himself, after 3 weeks of deep thought he announced that he “likes good coffee and looking at lakes”.

Photo 1.3: Phil at a conference. (Source: PD.)

Dr Marcus Gallagher is a Senior Lecturer in the School of Information Technology and Electrical Engineering. He did his undergrad in computer science at the University of New England and completed a PhD in Computer Science and Electrical Engineering at the University of Queensland in 2000. Since then he has worked at UQ as a Researcher and Academic.

His area of research is Artificial Intelligence, more specifically in machine learning and nature-inspired optimization algorithms. Broadly speaking, these algorithms are techniques for solving hard computational problems. He has collaborated with other researchers in applying these techniques to problems in astronomy and the analysis of health-care data.

When he used to have spare time, he enjoyed appropriately geeky activities, including reading science fiction novels, playing computer games and listening to heavy metal.

Photo 1.4: Marcus on a mountain. (Source: MGa.)

1.3 You

- SCIE1000 students come from many backgrounds, with diverse interests. Here is some information about the 546 students who took SCIE1000 in 2008; the cohort this year should be similar.
- 88.9% of students completed high school in Queensland, 5.7% elsewhere in Australia and 5.4% overseas (in China, Japan, South Korea, Saudi Arabia, Vietnam, Singapore, India, Malaysia, France, Malaysia, Sri Lanka, Vietnam, Hong Kong, Mexico, New Caledonia South Africa and Slovenia).
- 79.6% came directly from high school, 12.8% had a break of one year, and 7.6% longer.
- 30.7% had completed Maths C or equivalent, 68.8% Maths B and 0.5% Maths A.
- 62.2% of students were enrolled in a BSc, 21.9% in a BBiomedSc, 8.2% in a MBBS/BSc, 2.8% in a BBiotech, 2.4% in a BMarSt, 2.1% in a BSc/BA and 0.4% in a BSc/BEd.
- Students were asked to identify their primary area of scientific interest at the start and end of semester. The responses were:

Area	% at start	% at end
Biology	22.4%	29.0%
Biomedical Science	51.3%	39.2%
Chemistry	10.4%	7.0%
Computer Science	0.7%	0.6%
Earth Sciences	1.3%	1.5%
Geographical Sciences	0.7%	0.6%
Mathematics	2.4%	6.1%
Physics	4.1%	6.1%
Psychology	3%	4.6%
Other	3.7%	5.5%

- When asked to rate the importance of Mathematics to their area of science, on a scale of 5 (very important) to 1 (very unimportant), 30.7% of students responded 5, 49.6% responded 4, 11.3% responded 3, 2.8% responded 2 and 0.7% responded 1.
- When asked to rate the importance of Computing to their area of science on the same scale, 15.6% responded 5, 56.8% responded 4, 21.5% responded 3, 5.2% responded 2 and 0.9% responded 1.

1.4 Relationships

We believe that students and lecturers in a course incur a number of obligations, outlined below. Each party should inform the other if they believe that these obligations are not being met.

We will do our best to deliver a course that:

1. contains modern, interesting content from a range of science areas;
2. is relevant to your studies and future professions;
3. is intellectually challenging, accurate and correct;
4. is well-taught, by a team of engaging, professional experts;
5. respects your diverse backgrounds, aspirations and abilities;
6. helps you to improve both your technical knowledge and your generic learning skills;
7. includes assessment that is appropriate, challenging and identifies your level of skills, without being excessive; and
8. provides you with useful, appropriately timed feedback.

We expect that you will do your best to:

1. commit an appropriate amount of time, effort and intellectual engagement to your studies, and submit assessment on time;
2. attend lectures, tutorials and computer laboratory classes, and remain quiet and attentive in class;
3. respect your classmates, the teaching staff and the course content;
4. complete necessary pre-readings before lectures;
5. accept that at times we will cover content that you will find difficult, or of which you may not immediately see the relevance;
6. actively study all components of the course, including science, mathematics, computing and philosophy;
7. not plagiarise from classmates or other sources; and
8. seek help and advice in a timely manner.

1.5 Science

Science

Science aims to **understand**, **explain**, **predict** and **influence** phenomena. Understanding science, and thinking in a 'scientific manner', requires:

- *discipline knowledge and content* – the language, information, knowledge and skills specific to a discipline;
- *scientific thinking and logic* – the conceptual process of performing systematic investigations, hypothesising, thinking critically and defensibly, and making valid deducations and inferences;
- *communication and collaboration* – the process of working with others, sharing information and resources;
- *curiosity, creativity and persistence* – the relatively intangible characteristics that include the ability to ask and answer 'interesting' questions, and solve difficult problems in novel ways;
- *observation and data collection* – the the processes and techniques used to collect useful data about particular phenomena;
- *modelling and analysis* – the process of developing conceptual representations of phenomena, then using approximation, c mathematics, statistics and computation in order to allow predictions to be made.

These are combined in the "flower of science" on the front of these notes.

- Throughout your studies, different courses will develop different aspects of your science skills, which together allow you to graduate with a range of skills and knowledge necessary to understand and do science.
- Figure 1.1 illustrates the relative balance of science skills covered by various first-year courses. Make sure you appreciate what each course aims to achieve, and hence how your courses fit together and how they differ.

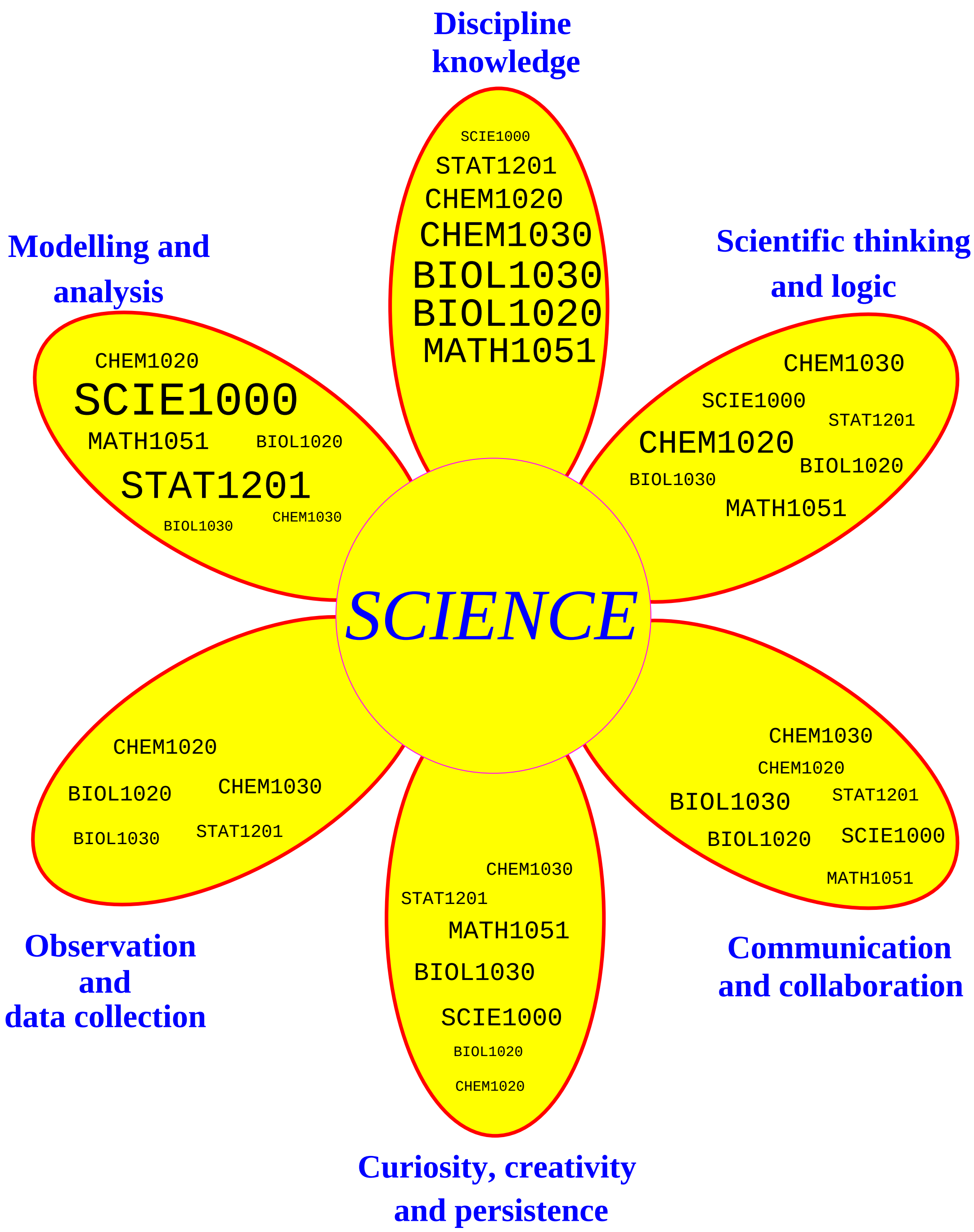


Figure 1.1: The relative balance of science skills covered by some first-year UQ science courses; the fontsize illustrates is proportional to the relative coverage in each course.

Chapter 2: **A career in modelling**

I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorem I'm teeming with a lot o' news,
With many cheerful facts about the square of the hypotenuse.
I'm very good at integral and differential calculus;
I know the scientific names of beings animalculous:
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.

Artist: Gilbert and Sullivan (www.youtube.com/watch?v=zSGWoXDFM64)

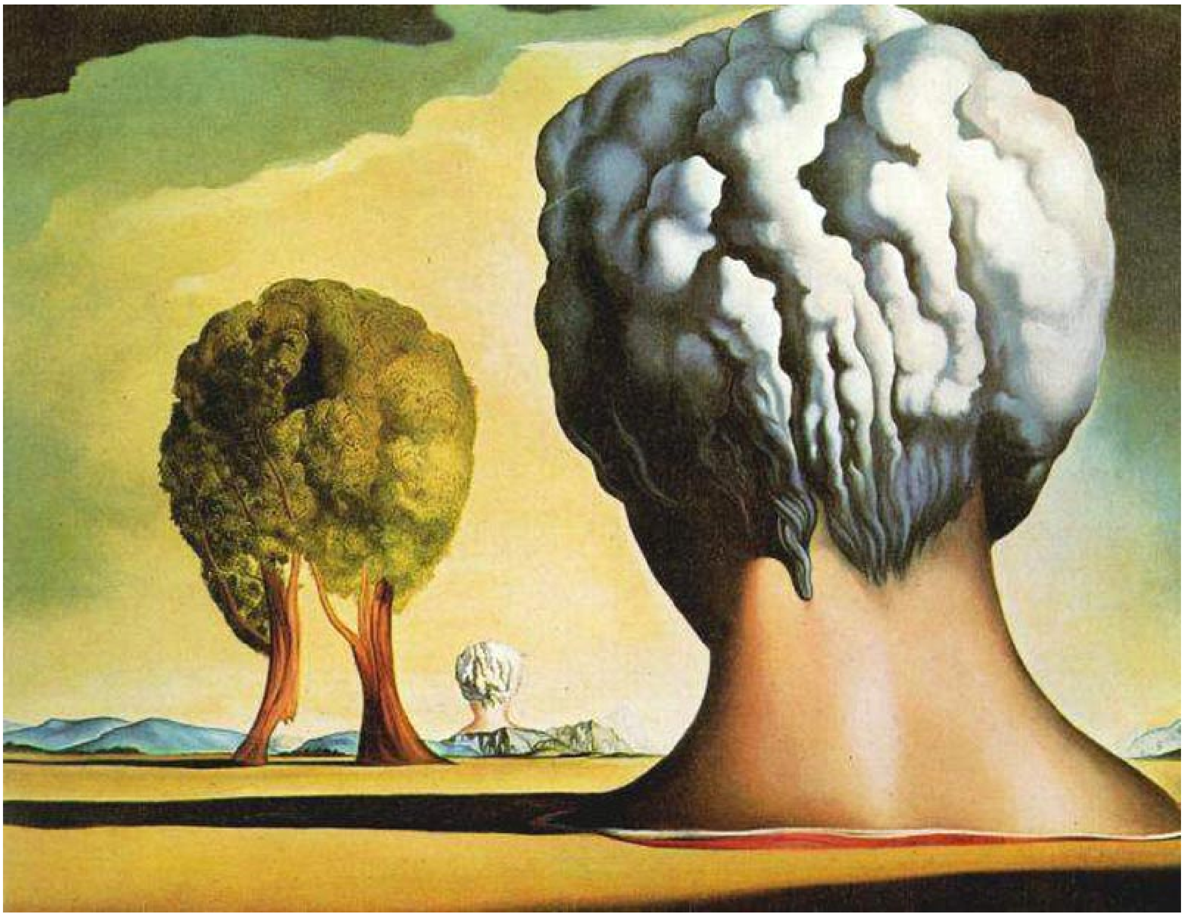


Image 2.1: *The Three Sphinxes of Bikini* (1947), Salvadore Dali (1904 – 1989), Morohashi Museum of Modern Art. (Source: Museum publication.)

2.1 Bloody alcohol

2.2 Science's next top model

- Earlier we stated that *science aims to understand, explain, predict and influence phenomena.*

- The concept of *change* (and the rate at which it occurs) is fundamental to science. (If we **know for certain** that something will not change then there is usually little interest in studying it.)

- Change can be naturally occurring or man-made, and desirable or undesirable.

- Most science is fundamentally quantitative, because quantifying phenomena allows us to measure, describe and compare variations in an efficient and precise manner.

- Science often involves observing and measuring values, such as the amount, frequency, magnitude, duration or rate of some phenomenon, then answering predictive questions about that phenomenon, such as
  - "What will happen if ...?"
  - "What causes ...?"
  - "How can we ...?"
  - "Why does ...?"

- A common approach is to use a *model,* based on the observed, measured data. Models are simplifications of the real world which allow us to:
  - make predictions about likely future events;
  - evaluate the possible impacts of interventions; and
  - investigate the robustness and stability of a phenomenon.

- Statistics is fundamental to the modelling process, allowing development of a theoretical model based on uncertain, imprecise data.

Models

Models aim to approximate reality, allowing the extraction of useful and meaningful conclusions about various events and processes, while at the same time being convenient and easy to use. All models need to strike a balance between accuracy and complexity.

The process of modelling

The process of modelling typically involves:

- **observing** some phenomenon;
- **thinking** about what relationships or patterns are important;
- **measuring** and recording data;
- **using** statistics to address uncertainty, imprecision and errors;
- **developing** equations to represent the data approximately;
- **using** mathematical techniques to simplify the equations;
- **writing** and executing computer models;
- **interpreting** results and relating them to the phenomenon;
- **comparing** modelled outcomes with actual outcomes;
- **refining** the model as required;
- **applying** the model using various conditions and assumptions;
- **predicting** possible future outcomes; and
- **communicating** results to an appropriate audience.

- Ways of developing 'appropriate' models include:
  - using "common sense" and logical deduction
  - using existing knowledge of similar phenomena; and
  - observing measured data and seeing what they "look like" (many phenomena change according to simple underlying patterns, such as at a constant rate or at a rate proportional to the current value).

Question 2.2.1

List some strengths and weaknesses of each of the five common ways of presenting quantitative models:

(a) Words

(b) Values (such as weight/height/age tables)

(c) Pictures (such as graphs)

(d) Mathematics

(e) Computer programs

- Note that there is nothing "right" or "wrong" about each approach – each is suited to different uses and/or target audiences. Most models can be developed and presented in **all** of these ways.
- In SCIE1000 we will use all five methods, but will focus on the final two.

2.3 Mathematics and models

- Some people believe that mathematics is an abstract process and is separate from science and the 'real world', unlike disciplines such as biology or chemistry that relate directly to the real world.
- These perceptions of mathematics and science are incorrect.
- Certainly, scientists use a combination of discipline knowledge and a special language to describe nature and the real world (for example, biologists use taxonomic categories, anatomical descriptions and medical terminology).
- Mathematicians also use a combination of discipline knowledge and a special language to describe nature and the real world (for example, exponential, linear and square root all describe relationships between observed data from natural phenomena).

Mathematics

Mathematics is a standardised formal language that allows us to:

- develop models to represent reality;
- perform correct, logical deductions;
- communicate unambiguously; and
- draw conclusions and make predictions.

- Whatever your area of science, you will need to learn the scientific language and knowledge that allows you to practise in that discipline.
- Similarly, because all areas of personal and professional life include quantitative concepts, everyone needs to learn the mathematical language and knowledge that allows them to live and work.
- Studying and working in more specialised areas (such as science) requires a higher level of mathematical knowledge, and sophistication in its use.

- SCIE1000 uses mathematical language and knowledge, but we do not study mathematics for its own sake, or to develop new mathematical knowledge; if you wish to do that then enrol in discipline-based mathematics courses.
- Instead, we study mathematics **solely** for its fundamental role in describing and modelling the real world, and we will interpret mathematical language in this context. For example:
  - **Statistics** is the process of addressing uncertainty, imprecision and errors in data, while allowing trends and patterns within the data to be observed and deciphered.
  - The **mathematical function** is the formal representation of a pattern in a collection of values.
  - **Logical deduction** describes the process of starting with a collection of facts, approximations and knowledge, and then following a sequence of logically defensible steps that leads to valid conclusions or deductions.
- Sometimes we cannot directly measure a phenomenon of interest (due to physical, ethical or financial limitations). Instead, we may be able to measure and model a related phenomenon.
- We can then model the (unmeasurable) quantity using analytical techniques such as:
  - **algebra**, which allows us to conduct logically valid manipulations, simplifications and transformations;
  - **differentiation**, which allows us to model an (unmeasurable) rate of change in a (measurable) phenomenon; and
  - **integration**, which allows us to model an (unmeasurable) phenomenon based on a (measurable) rate of change.
- Mathematics gives us a range of logical and valid techniques that allow us to deduce information that we cannot measure or obtain in other ways!

Science and mathematics are not separate areas, with mathematics *artificial* or *irrelevant*. Instead, they are often so closely interlinked that they are indistinguishable!

2.4 Modelling in action

Case Study 1:
Let it flow

- *Fluid dynamics* involves studying liquids and gases that are moving, a process that is important in many branches of science (particularly geology, environmental science and biomedical science) and engineering.

Question *2.4.1*

Develop a model of the flow rate of blood through a given blood vessel. (Hint: which factors are important; do they increase or decrease the rate?)

The following formula (called the *Hagen-Poiseuille equation*) is often used to estimate such flows:

Compare your formula with the Hagen-Poiseuille equation.

- High levels of certain types of cholesterol in the blood can lead to blockages in coronary arteries, which in turn can lead to a heart attack.
- During a heart attack, heart muscle tissue dies and is replaced by scar tissue.

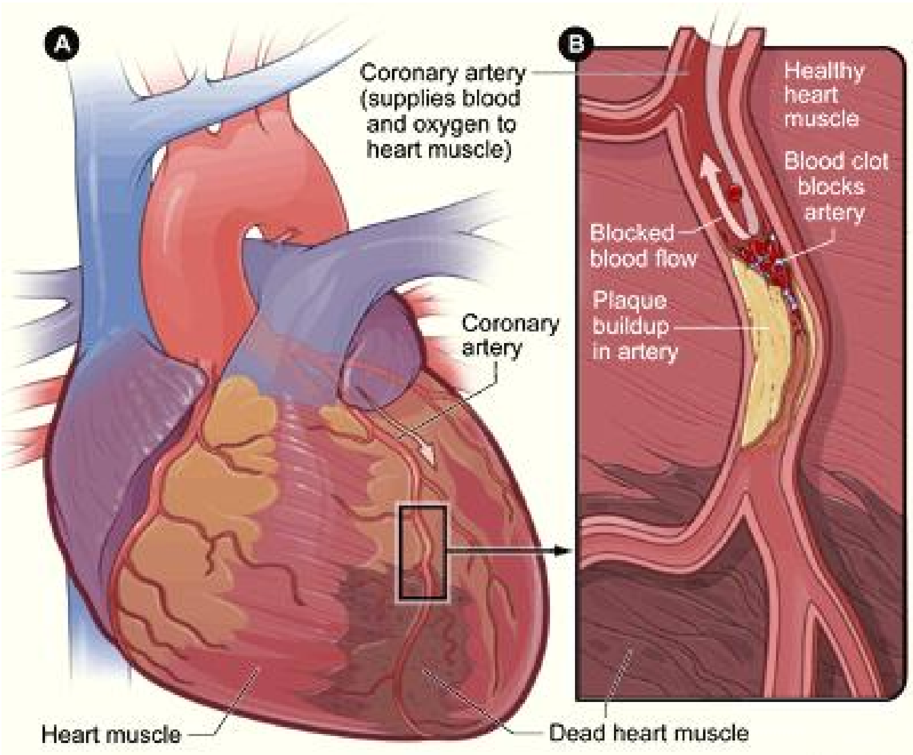


Figure 2.1: Left: heart and coronary artery showing dead heart muscle caused by a heart attack. Right: longitudinal section of a coronary artery with plaque buildup and a blood clot. (Source: www.nhlbi.nih.gov.)

- One surgical method of increasing blood flow through partially blocked arteries is an *angioplasty*.
- In a coronary angioplasty, a balloon-tipped catheter is inserted under local anaesthetic, typically through the groin or arm.
- When in position in the coronary artery, the balloon is inflated to expand the blood vessel (and sometimes a metallic stent is inserted to maintain the expansion).
- Angioplasties are much simpler and less invasive than coronary artery bypass surgery, but have a higher rate of recurrence of the original occlusion.

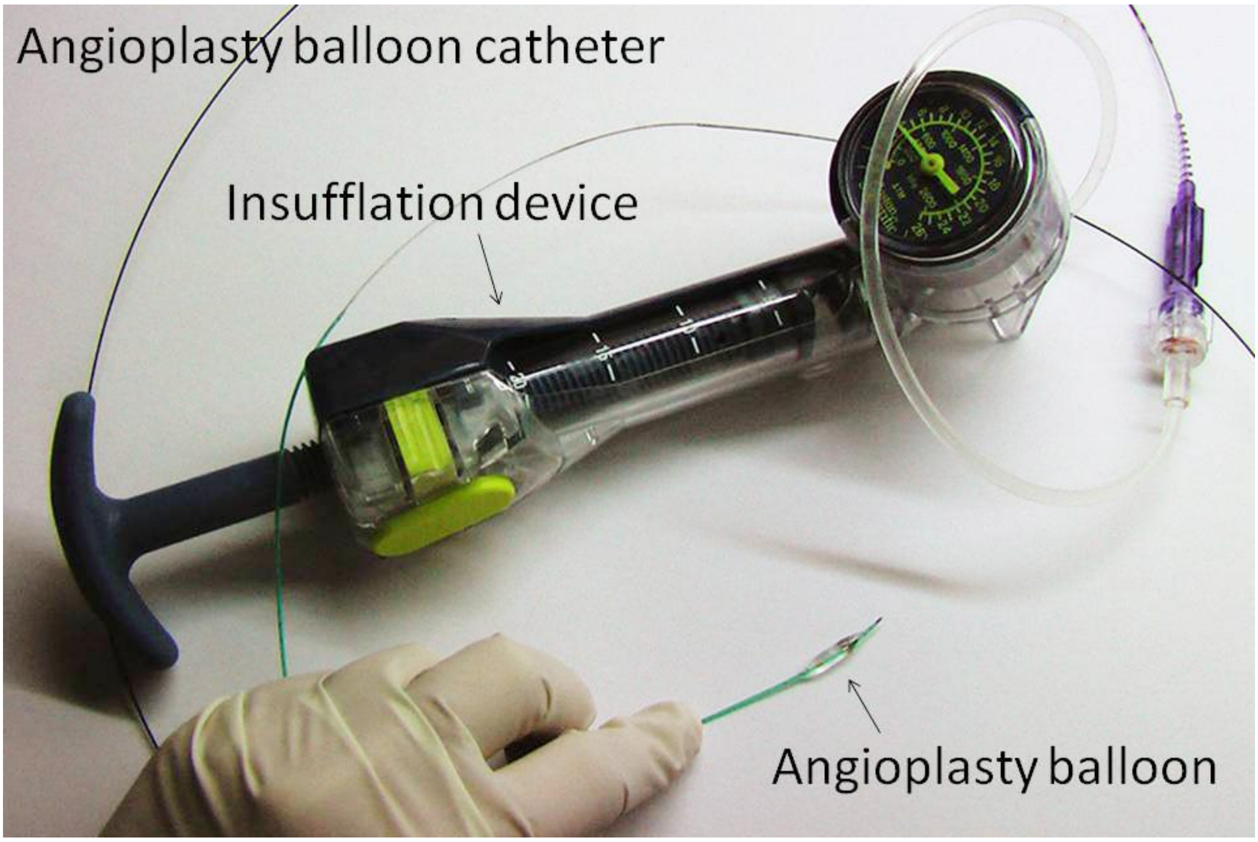


Photo 2.1: Angioplasty balloon catheter. (Source: DM.)

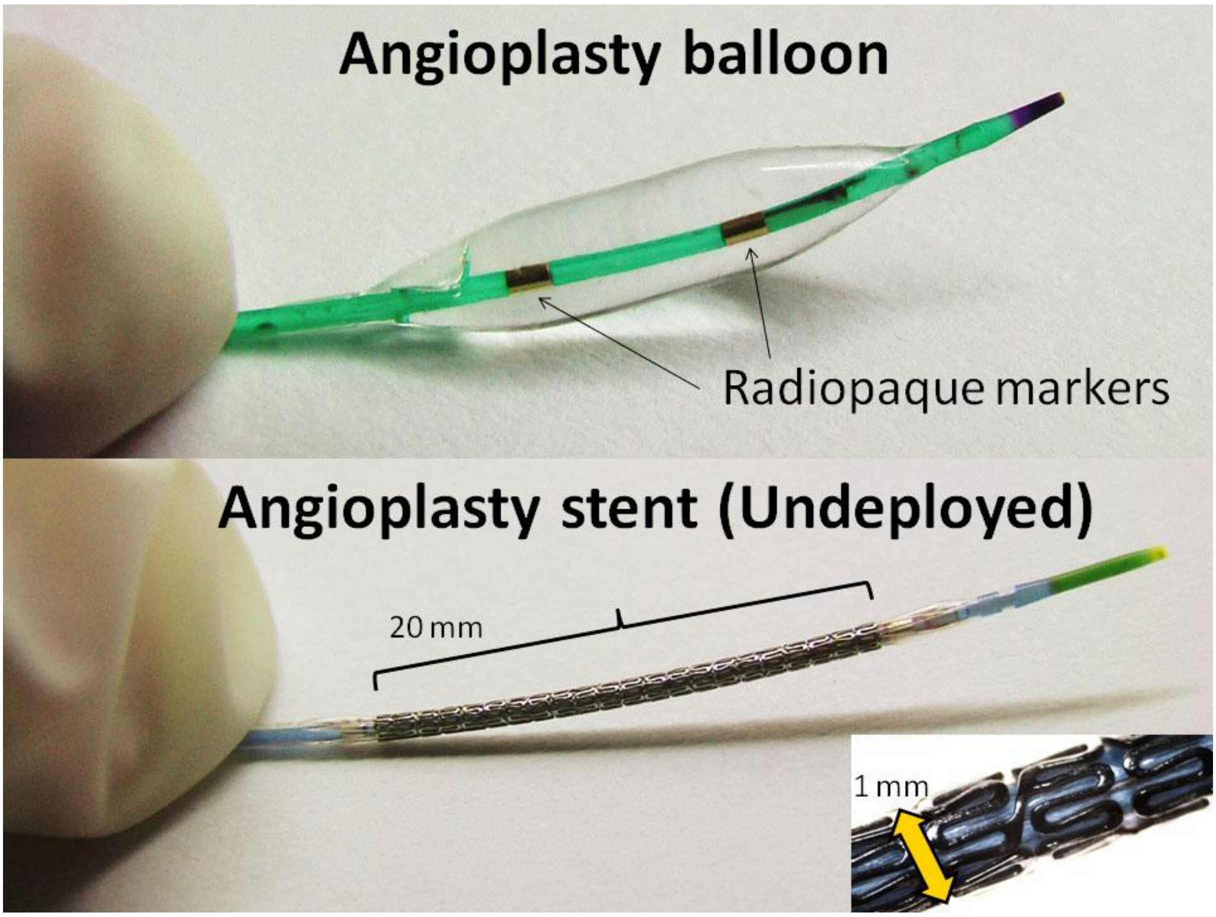


Photo 2.2: Inflated angioplasty balloon and undeployed stent. (Source: DM.)

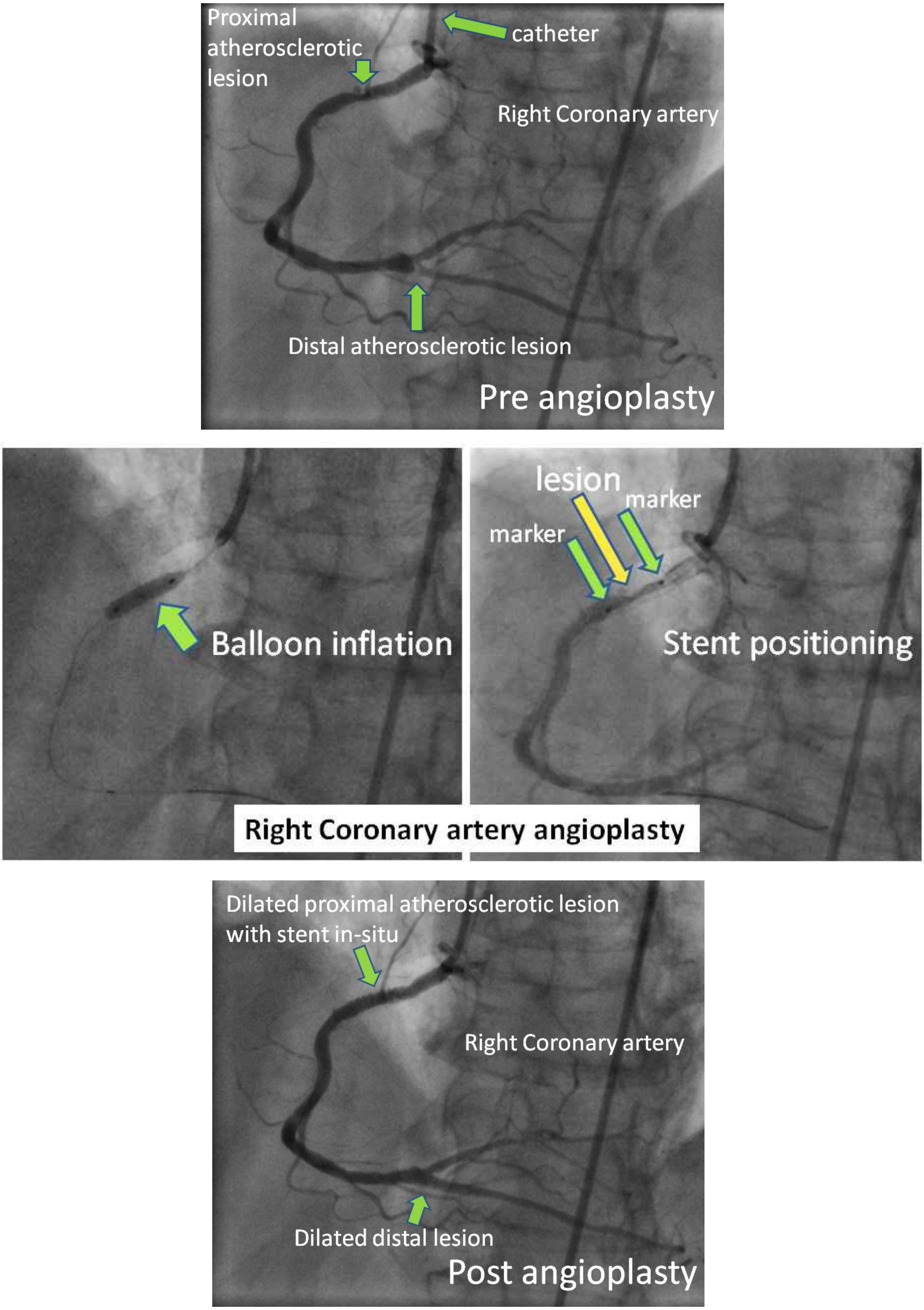


Photo 2.3: Right coronary artery angioplasty. (Source: DM.)

Question 2.4.2

Assume that a patient undergoing an angioplasty procedure shows a 30% increase in the diameter of a partially occluded artery. Estimate the resulting percentage increase in blood flow rate through that artery, and interpret your answer.

End of Case Study 1.

Case Study 2: Modelling the risk of heart disease

- Diseases of the circulatory system (including heart disease and stroke) are the leading cause of death in many western societies.

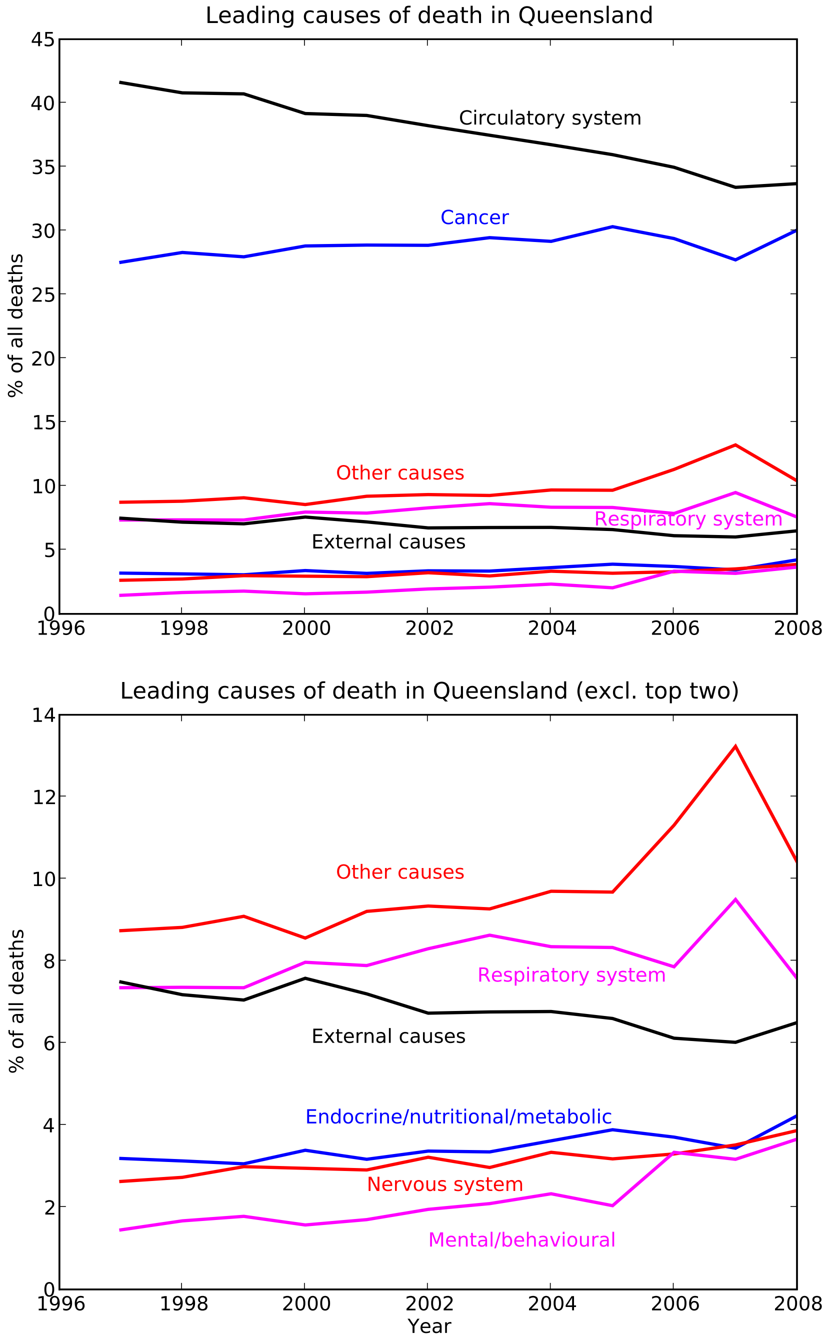


Figure 2.2: Leading causes of death in Queensland (data source: Qld government).

- Individuals, doctors and public health bodies all have an obvious interest in predicting the risk of suffering cardiovascular disease.

- In medicine and population health, risks are often specified as a probability of an identified event occurring in a given time period.
- Shortly we will encounter a famous, long-running study into cardiovascular health, called the *Framingham study*[1]. The study defines Coronary Heart Disease (CHD) as including:
  - *angina pectoris*, which is severe chest pain caused by a lack of blood to heart muscle;
  - *myocardial infarction*, commonly called a heart attack, arising from complete loss of blood supply to heart muscle; and
  - death due to cardiac arrest.
- CHD is most often caused by *atherosclerosis*, which is a blockage of a coronary artery supplying blood to heart muscle tissue.
- Photograph 2.4 shows a calf heart, with coronary arteries clearly visible.

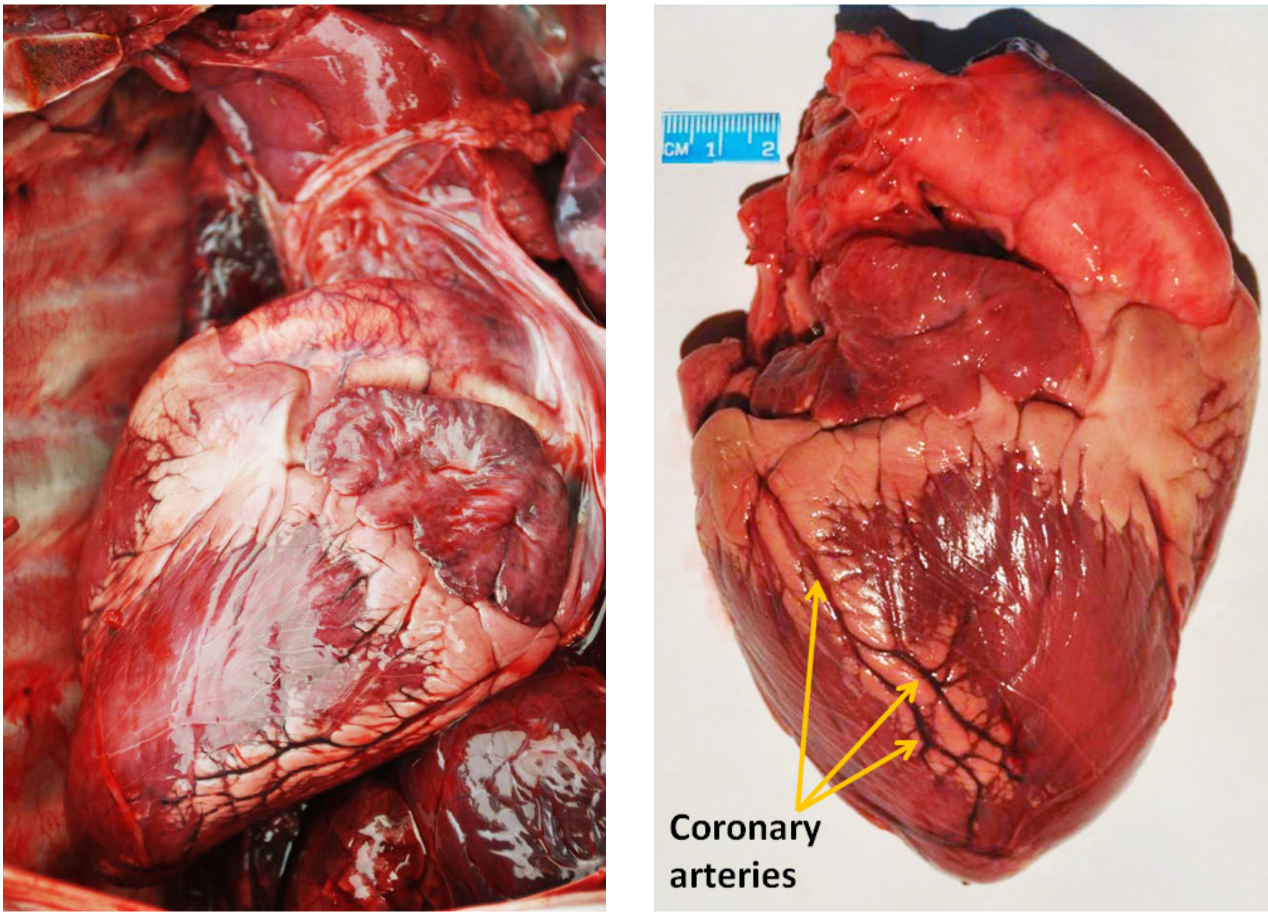


Photo 2.4: Left: calf heart in-situ. Right: calf heart showing coronary arteries. (Source: PA.)

[1]All information from the Framingham study has been reproduced with permission from the National Heart, Lung, and Blood Institute as a part of the National Institutes of Health and the U.S. Department of Health and Human Services.

Question 2.4.3

Which factors or data are crucial when developing a model for estimating the likelihood that a person will suffer CHD in the next 10 years? Does each factor increase or decrease the risk?

What is your "gut feeling" of the likelihoods that POD and Peter will suffer CHD in the next 10 years?

- Until comparatively recently, little was known about the general causes of heart disease and stroke, although the rates of cardiovascular disease (CVD) in many societies had been rising for some time.
- In 1948, a study into heart disease commenced in Framingham, Massachusetts, which has become one of the best-known longitudinal health studies.
- The Framingham study (which continues today) has monitored the cardiovascular health of the participants, identified a range of risk factors and included them in a mathematical risk model.

***Extension** 2.4.4 (from [14])*

"Over the years, careful monitoring of the Framingham Study population has led to the identification of the major CVD risk factors – high blood pressure, high blood cholesterol, smoking, obesity, diabetes, and physical inactivity – as well as a great deal of valuable information on the effects of related factors such as blood triglyceride and HDL cholesterol levels, age, gender, and psychosocial issues.... Since its inception, the study has produced approximately 1,200 articles in leading medical journals...Research milestones from the Framingham study include:

- 1960: Cigarette smoking found to increase the risk of heart disease
- 1961: Cholesterol level, blood pressure, and electrocardiogram abnormalities found to increase the risk of heart disease
- 1967: Physical activity found to reduce the risk of heart disease and obesity to increase the risk of heart disease
- 1970: High blood pressure found to increase the risk of stroke
- 1976: Menopause found to increase the risk of heart disease
- 1978: Psychosocial factors found to affect heart disease
- 1988: High levels of HDL cholesterol found to reduce risk of death
- 1996: Progression from hypertension to heart failure described
- 1998: Development of simple coronary disease prediction algorithm involving risk factor categories to allow physicians to predict multivariate coronary heart disease risk in patients without overt CVD
- 1999: Lifetime risk at age 40 years of developing coronary heart disease is one in two for men and one in three for women
- 2001: High-normal blood pressure is associated with an increased risk of cardiovascular disease.
- 2002: Lifetime risk of developing high blood pressure in middle-aged adults is 9 in 10.
- 2002: Obesity is a risk factor for heart failure.

- One of the resources produced from the Framingham Study is a CHD Risk Prediction score sheet, used to predict the likelihood that a person will suffer CHD in the next ten years.

Step 1: Age

Age (Years)	Points Female	Points Male
30-34	-9	-1
35-39	-4	0
40-44	0	1
45-49	3	2
50-54	6	3
55-59	7	4
60-64	8	5
65-69	8	6
70-74	8	7

Step 2: LDL cholesterol

LDL (mmol/L)	Points Female	Points Male
≤ 2.59	-2	-3
2.60-3.36	0	0
3.37-4.14	0	0
4.19-4.91	2	1
≥ 4.92	2	2

Key	
Colour	Risk
green	very low
white	low
yellow	moderate
rose	high
red	very high

Step 3: HDL cholesterol

HDL (mmol/L)	Points Female	Points Male
≤ 0.9	5	2
0.91-1.16	2	1
1.17-1.29	1	0
1.30-1.55	0	0
≥ 1.56	-2	-1

Step 4: Blood pressure (F: Female, M: Male)

Systolic (mm Hg)	Diastolic (mm Hg)				
	< 80 Points	80-84 Points	85-89 Points	90-99 Points	≥ 100 Points
≤ 120	F: -3 M: 0				
120-129		0			
130-139			F: 0 M: 1		
140-159				2	
≥ 160					3

Note: When systolic and diastolic pressures provide different estimates for point scores, use the higher number.

Step 5: Diabetes

Diabetes	Points Female	Points Male
No	0	0
Yes	4	2

Step 6: Smoker

Smoker	Points Female	Points Male
No	0	0
Yes	2	2

Step 7: Sum points from Steps 1-6

Category	Points
Age	
LDL	
HDL	
Blood pressure	
Diabetes	
Smoker	
Point total	

Step 8: Determine risk from point total

Point total	10 Year CHD risk Female	10 Year CHD risk Male
≤ -3	1%	1%
-2	1%	2%
-1	2%	2%
0	2%	3%
1	2%	4%
2	3%	4%
3	3%	6%
4	4%	7%
5	5%	9%
6	6%	11%
7	7%	14%
8	8%	18%
9	9%	22%
10	11%	27%
11	13%	33%
12	15%	40%
13	17%	47%
14	20%	≥ 56%
15	24%	≥ 56%
16	27%	≥ 56%
≥ 17	≥ 32%	≥ 56%

Step 9: Compare to others of the same age

Age (Years)	Average 10 Yr risk	Low 10 Yr risk
30-34	F: <1% M: 3%	F: <1% M: 2%
35-39	F: 1% M: 5%	F: <1% M: 3%
40-44	F: 2% M: 7%	F: 2% M: 4%
45-49	F: 5% M: 11%	F: 3% M: 4%
50-54	F: 8% M: 14%	F: 5% M: 6%
55-59	F: 12% M: 16%	F: 7% M: 7%
60-64	F: 12% M: 21%	F: 8% M: 9%
65-69	F: 13% M: 25%	F: 8% M: 11%
70-74	F: 14% M: 30%	F: 8% M: 14%

Note: low risk was calculated for an individual of the same age, with normal blood pressure, LDL 2.60-3.36 mmol/L, HDL 1.45 mmol/L, non-smoker and no diabetes.

Figure 2.3: Framingham CHD risk assessment sheet for males and females

Question 2.4.5

Use the Framingham CHD risk assessment sheet in Figure 2.3 to estimate the probability that Peter and POD will suffer CHD within 10 years. Compare this with your answers to Question 2.4.3.

Question 2.4.6

Briefly discuss some key points highlighted by the risk prediction sheet. (You may like to mention such things as the comparative impact of different risk factors, some 'risk factors' commonly mentioned in the media that are not included, and some differences between males and females.)

Question 2.4.7

How could you assess the **accuracy** and **usefulness** of the Framingham CHD risk assessment model?

Extension *2.4.8 (from [4])*

The Framingham risk 'score' sheet is an approximate, table-based representations of a mathematical model of CHD risk. The mathematical model for males is as follows.

Let

- b = systolic blood pressure (mm Hg)
- t = total blood cholesterol (mmol/L)
- h = high density lipoprotein cholesterol (mmol/L)
- a = age (years)
- s = smoker? (0=no, 1=yes)
- g = electrocardiograph left ventricular hypertrophy? (0=no, 1=yes)
- d = diabetes? (0=no, 1=yes)

Calculate

- $\mu_1 = 15.5303 - 0.9119 \ln b - 0.2767s - 0.7181 \ln(t/h)$
- $\mu_2 = -0.5865g - 1.4792 \ln a - 0.1759d$
- $\mu = \mu_1 + \mu_2$
- $\sigma = e^{-0.3155 - 0.2784(\mu - 4.4181)}$
- $u = \dfrac{\ln 10 - \mu}{\sigma}$

Then the value

$$p = 1 - e^{-e^u}$$

gives the estimated probability that a male with the given characteristics will suffer a CHD event in the next ten years of his life.

- The above risk equation can be implemented in a Python program.

Program 2.1: 10-year CHD risk calculator for males

```
# 10-year risk of CHD for a male.
from pylab import *

# Input relevant data
print("Please enter the following:")
b = eval(input("systolic blood pressure (mm Hg) "))
t = eval(input("total blood cholesterol (mmol/L) "))
h = eval(input("HDL cholesterol (mmol/L) "))
a = eval(input("age (years) "))
s = eval(input("smoker? (0=no, 1=yes) "))
g = eval(input("ECG left ventr. hypertrophy? (0=no, 1=yes) "))
d = eval(input("diabetes? (0=no, 1=yes) "))

# Calculate risk
mu1 = 15.5303 - 0.9119*log(b) - 0.2767*s - 0.7181*log(t/h)
mu2 = -0.5865*g - 1.4792*log(a) - 0.1759*d
mu = mu1 + mu2
sigma = exp(-0.3155 - 0.2784*(mu - 4.4181))
u = (log(10) - mu)/sigma

p = 1-exp(-exp(u))
risk = round(100*p)
print("The 10-year probability of a CHD event is ",risk,"%")
```

Here is the output from running this program.

```
Please enter the following:
systolic blood pressure (mm Hg) 120
total blood cholesterol (mmol/L) 4.7
HDL cholesterol (mmol/L) 0.9
age (years) 46
smoker? (0=no, 1=yes) 0
ECG left ventr. hypertrophy? (0=no, 1=yes) 0
diabetes? (0=no, 1=yes) 0
The 10-year probability of a CHD event is  7.0 %
```

End of Case Study 2.

Case Study 3: Get your BAC up

- Blood Alcohol Concentration (BAC) is usually measured as the percentage of total blood volume which is alcohol.
- Figure 2.4 (see [41]) shows some physical and behavioural effects of alcohol consumption typically experienced by people at different BAC levels.
- There are strict laws about driving and operating machinery after consuming alcohol.
- In Australia the maximum legal BAC for driving is 0.05%, or 0.5 g/L.

Stages	BAC	Likely Effects
Feeling of well-being	Up to .05%	Talkative; Relaxed More confident
At-risk	.05−.08%	Talkative Acts and feels self-confident Judgment and movement impaired Inhibitions reduced
Risky state	.08−.15%	Speech slurred Balance and coordination impaired Reflexes slowed Visual attention impaired Unstable emotions Nausea, vomiting
High-risk state	.15−.30%	Unable to walk without help Apathetic, sleepy Laboured breathing Unable to remember events Loss of bladder control Possible loss of consciousness
Death	Over .30%	Coma; Death

Figure 2.4: Typical physical and behavioural effects of alcohol at various BAC levels.

Extension *2.4.9 (from [41])*

'**Intoxication risks**: Intoxication is the most common cause of alcohol-related problems, leading to injuries and premature deaths. As a result, intoxication accounts for two-thirds of the years of life lost from drinking. Alcohol is responsible for:

- 30% of road accidents
- 34% of falls and drownings
- 12% of suicides
- 44% of fire injuries
- 16% of child abuse cases
- 10% of industrial accidents

As well as deaths, short-term effects of alcohol result in illness and loss of work productivity (e.g. hangovers, drink driving offences). In addition, alcohol contributes to criminal behaviour – in Australia over 70% of prisoners convicted of violent assaults have drunk alcohol before committing the offence and more than 40% of domestic violence incidents involve alcohol.

Long-term effects: Each year approximately 3000 people die as a result of excessive alcohol consumption and around 101 000 people are hospitalised. Long-term excessive alcohol consumption is associated with:

- heart damage
- high blood pressure and stroke
- liver damage
- cancers of the digestive system
- other digestive system disorders (e.g. stomach ulcers)
- sexual impotence and reduced fertility
- increasing risk of breast cancer
- sleeping difficulties
- brain damage with mood and personality changes
- concentration and memory problems "

- It is often useful to be able to estimate the time for BAC to return to 0. The time will vary somewhat between individuals, but governments and health bodies publish general guidelines.

Question 2.4.10

Figure 2.5 shows the approximate times required for the BAC of males of different masses to return to 0. (The term *weight* would be commonly used, but *mass* is the technically more correct.) Derive a mathematical model for the data in Figure 2.5.

num.	Mass (pounds)							
drinks	120	140	160	180	200	220	240	260
1	2	2	2	1.5	1	1	1	1
2	4	3.5	3	3	2.5	2	2	2
3	6	5	4.5	4	3.5	3.5	3	3
4	8	7	6	5.5	5	4.5	4	3.5
5	10	8.5	7.5	6.5	6	5.5	5	4.5

Figure 2.5: Approximate time (in hours) for BAC to return to 0 for males of different masses.

(Masses are given in pounds; to convert approximately from pounds to kg, divide by 2.2.)

Question 2.4.11

Briefly discuss the effectiveness and accuracy of the mathematical model. The actual and modelled times are given in Figure 2.6.

	Actual								Model							
num.	Mass (pounds)								Mass (pounds)							
drinks	120	140	160	180	200	220	240	260	120	140	160	180	200	220	240	260
1	2	2	2	1.5	1	1	1	1	2	1.7	1.5	1.3	1.2	1.1	1.0	0.9
2	4	3.5	3	3	2.5	2	2	2	4	3.4	3.0	2.7	2.4	2.2	2.0	1.8
3	6	5	4.5	4	3.5	3.5	3	3	6	5.1	4.5	4.0	3.6	3.3	3.0	2.8
4	8	7	6	5.5	5	4.5	4	3.5	8	6.9	6.0	5.3	4.8	4.4	4.0	3.7
5	10	8.5	7.5	6.5	6	5.5	5	4.5	10	8.6	7.5	6.7	6.0	5.5	5.0	4.6

Figure 2.6: Actual and modelled times for BAC to return to zero.

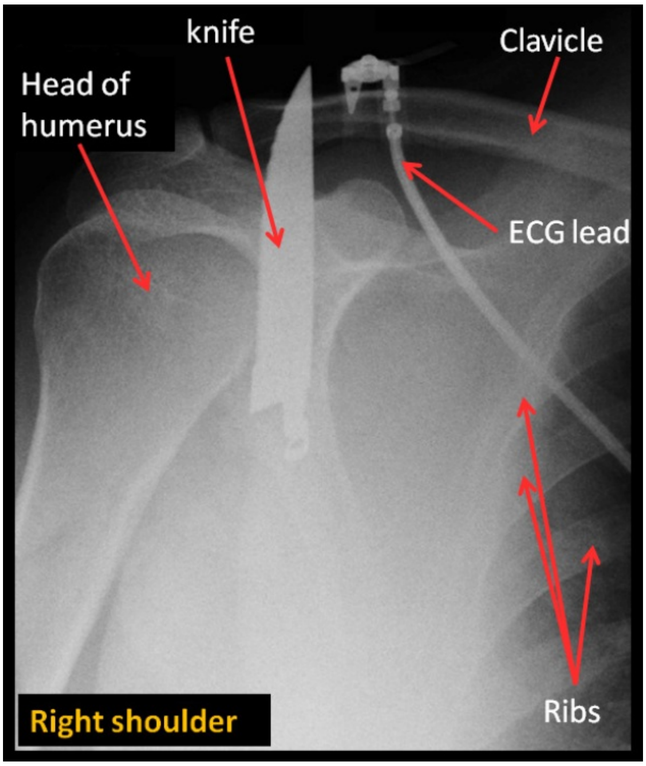


Photo 2.5: A sharp, stabbing pain in the right shoulder! (Source: Qld Health and DM.)

End of Case Study 3.

2.5 Units

- When measuring a physical quantity, modelling some phenomenon or communicating a result, it is essential to use a standard *unit of measurement*. The consequences of using inconsistent units can be severe.

***Example** 2.5.1*

The Mars Climate Orbiter was launched in 1998 as part of a $USD330 million project, but in September 1999 it crashed into Mars. Here is an extract from the report into the accident [29]:

> During the 9-month journey from Earth to Mars, propulsion maneuvers were periodically performed to remove angular momentum buildup in the on-board reaction wheels... The increased AMD events coupled with the fact that the angular momentum (impulse) data was in English, rather than metric, units, resulted in small errors being introduced in the trajectory estimate over the course of the 9-month journey. At the time of Mars insertion, the spacecraft trajectory was approximately 170 km lower than planned...
>
> ...it was discovered that the small forces ΔVs reported by the spacecraft engineers for use in orbit determination solutions was low by a factor of 4.45 (1 pound force = 4.45 Newtons) because the impulse bit data contained in the AMD file was delivered in lb-sec instead of the specified and expected units of Newton-sec.

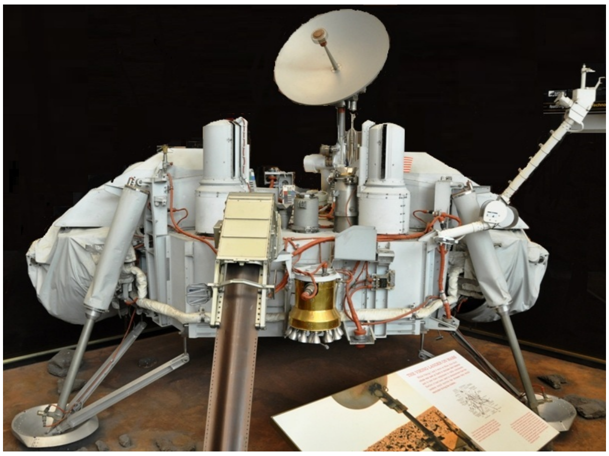


Photo 2.6: Mars Lander (proof test model) from the Viking program, launched 1975. (Source: PA.)

SI Units

The most commonly used units of measurement are defined by the **International System of units**, and are called **SI** units. There are seven **SI base units**; their standard names and symbols are shown in Figure 2.7.

Base quantity	SI unit name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Figure 2.7: Names and symbols of the seven SI base units.

SI prefixes

Figure 2.8 shows the 20 **SI prefixes** used to denote multiples of the SI units. Each prefix is a positive or negative power of 10.

Multiple	Name	Symbol	Multiple	Name	Symbol
10^1	deka	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Figure 2.8: The 20 SI prefixes.

Derived units

Many natural and scientific quantities require more complex units than SI base units. These **can always be defined** in terms of the seven base units, and are called **SI derived units**.

Example 2.5.2

Some examples of quantities with SI derived units are:

- Volume, measured in *cubic metres*;
- Concentration, measured in *moles per cubic metre*.

  (Concentration is often expressed as *moles per litre*. However, a *litre* (L) is defined to be 1/1000 of a *cubic metre*.)

Mathematical notation for SI derived units

There is a convenient standard mathematical notation for SI derived units, based on the following principles:

- if the quantity involves the mathematical "product" of two SI units then their SI symbols are separated by a space or a dot;
- mathematical power notation is used if the same SI unit occurs in a "product" more than once; and
- if the quantity involves the "quotient" of an SI unit then the derived unit either uses a quotient sign /, or (more often) mathematical power notation with a negative power.

Example 2.5.3

The quantities from Example 2.5.2 can be rewritten as:

- Volume, measured in m^3 (or L, where 1 L is defined to be 10^{-3} m^3).
- Concentration, measured in mol/L or $\mathrm{mol\ L}^{-1}$ or $\mathrm{mol} \cdot \mathrm{L}^{-1}$.

Example 2.5.4

Some SI derived units are used very frequently, so they have been given special names and symbols. Figure 2.9 shows some well-known examples.

Quantity	Name	Symbol	SI units	SI base units
frequency	hertz	Hz	-	s^{-1}
force	newton	N	-	$m \cdot kg \cdot s^{-2}$
pressure, stress	pascal	Pa	$N \cdot m^{-2}$	$m^{-1} \cdot kg \cdot s^{-2}$
energy, work, quantity of heat	joule	J	$N \cdot m$	$m^2 \cdot kg \cdot s^{-2}$
power, radiant flux	watt	W	$J \cdot s^{-1}$	$m^2 \cdot kg \cdot s^{-3}$
quantity of electricity, electric charge	coulomb	C	-	$s \cdot A$
electric potential difference, electromotive force	volt	V	$W \cdot A^{-1}$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
Celsius temperature	degree Celsius	$^\circ C$	-	K

Figure 2.9: Some well-known units and their SI base units.

Dimensional analysis

A useful technique in science is **dimensional analysis**, which is closely related to SI units. Some useful points are:

- Any equation describing a physical situation can only be true if it is **dimensionally homogeneous**; that is, both sides of the equation must have the same units.
- Units can be mathematically manipulated, including multiplied and cancelled.
- Quantities can be added or subtracted if, and only if, they have the same units.

Dimensional analysis enables a quick check of whether a calculation is 'plausible': if the dimensions do not match, then there must be an error.

The importance of units

Every physical quantity must have units unless it is a pure number (such as 2 or π). Every length must be measured in m, km, inches, furlongs, or some other unit of length. So if $x = 3$ m then x is a length, but if $y = 3$ then y is just a number. **These two things are different.**

In scientific work, you should always deal correctly with units. Sometimes, when learning new mathematical concepts, it can make things seem more complicated or difficult to read if units are included. To keep things simple, in SCIE1000 we often define variables to not require units. For example, consider the the following alternate definitions.

- "Let t be the time since the rocket was launched"; and
- "Let t be the number of seconds since the rocket was launched".

Both definitions are perfectly correct. If the first definition is used, then t requires a unit throughout any ensuing calculations. If the second is used, then t does not require a unit. The latter terminology is often used in these notes.

2.6 Space for additional notes

Chapter 3: **Introduction to python programming**

The coiling is fast
This time it's your last
Your soul asphyxiated
Final chance for escape terminated.
Enveloped in python
constriction complete
where dreams become nightmares
of total defeat.

Artist: Torniquet (www.youtube.com/watch?v=c107Aor329g)

Image 3.1: *Venus, Cupid, Folly and Time*, (1540 – 45), Agnolo di Cosimo (usually known as Il Bronzino) (1503 – 1572), National Gallery, London. (Source: en.wikipedia.org)

3.1 Python

- Earlier we encountered five ways of representing models in science; computational models were formed one of these categories.
- Computation is important when formulating and applying models, particularly when dealing with complex phenomena.
- Every computer program and computer model must be implemented in some computer *language*.
- A computer language is a collection of commands that can be interpreted by a computer, instructing the computer to perform associated operations and calculations.
- There are many different computer languages, each suited to various uses. In SCIE1000 we use the language *Python*. (For interest, Python was named after *Monty Python's Flying Circus*.)
- We use Python because it is modern, freely available, fairly easy to learn, used in real science applications, and illustrates many important general computing concepts.
- Python users include Youtube, Google, Yahoo!, CERN and NASA.

Python in SCIE1000

You will encounter Python in SCIE1000 in the following ways:

- These lecture notes include some examples of Python programs and their output.
- You have a separate Python programming manual.
- You will write small Python programs in your computer lab classes and submit some of them for assessment.

You will **not** need to write programs in your exam. However, you will need to answer questions on general computing concepts, and also explain what some given Python programs do.

3.2 Introduction to programming

- Software design and development is a huge industry.
- Much of modern life relies on computer software, including cars, planes, payroll systems, phone systems, hospitals, education and defence.
- In SCIE1000, we will only write short, relatively simple programs to model some phenomena. However, even when programs are not complex, it is still important to use good programming techniques.
- As you write programs, you should always be guided by a number of "good programming" principles.

Good programming

There are many features of a "good" computer program. In general, programs should be:

- correct;
- easy to read;
- easy to understand;
- simple;
- efficient;
- thoroughly tested;
- well-documented; and
- easy to use.

- To assist with achieving these goals, programs should:
  - include blank lines and spacing to assist readability;
  - have extensive comments to explain what is happening; and
  - use *meaningful names* for variables and functions.

- Earlier we stated that "a computer language is a collection of commands that can be interpreted by a computer, and instructs the computer to perform associated operations and calculations".
- Different languages provide different commands, however the following types of command are typically part of many languages. (A brief Python example follows each command.)
  - Comments – these are ignored by the computer, but make programs easier to understand.

    In Python programs, lines commencing with `#` are comments.
  - Input commands – these allow data to be entered into the program from the keyboard or a file.

    In Python, the command `input` reads data from the keyboard.
  - Output commands – these allow data to be displayed on the screen or sent to a file.

    In Python, the command `print` displays text and `plot` draws graphs.
  - Variables – these allow data values to be stored and manipulated.

    Python allows variables which store single data values, and also variables called *arrays* that store multiple data values.
  - Calculations – these allow the computer to perform a range of mathematical calculations.

    Python supports all standard mathematical calculations.
  - Booleans – these allow the computer to evaluate expressions as being true or false.

    Python uses values `True` and `False`, and words such as `and`, `or`.
  - Conditional execution – these allow the computer to execute certain commands if, and only if, a boolean expression is true.

    Python provides the conditional command `if-then-else`.
  - Loops – these allow the computer to execute commands multiple times, while a boolean expression is true.

    Python provides a number of loops, including `while` loops.

3.3 Designing programs

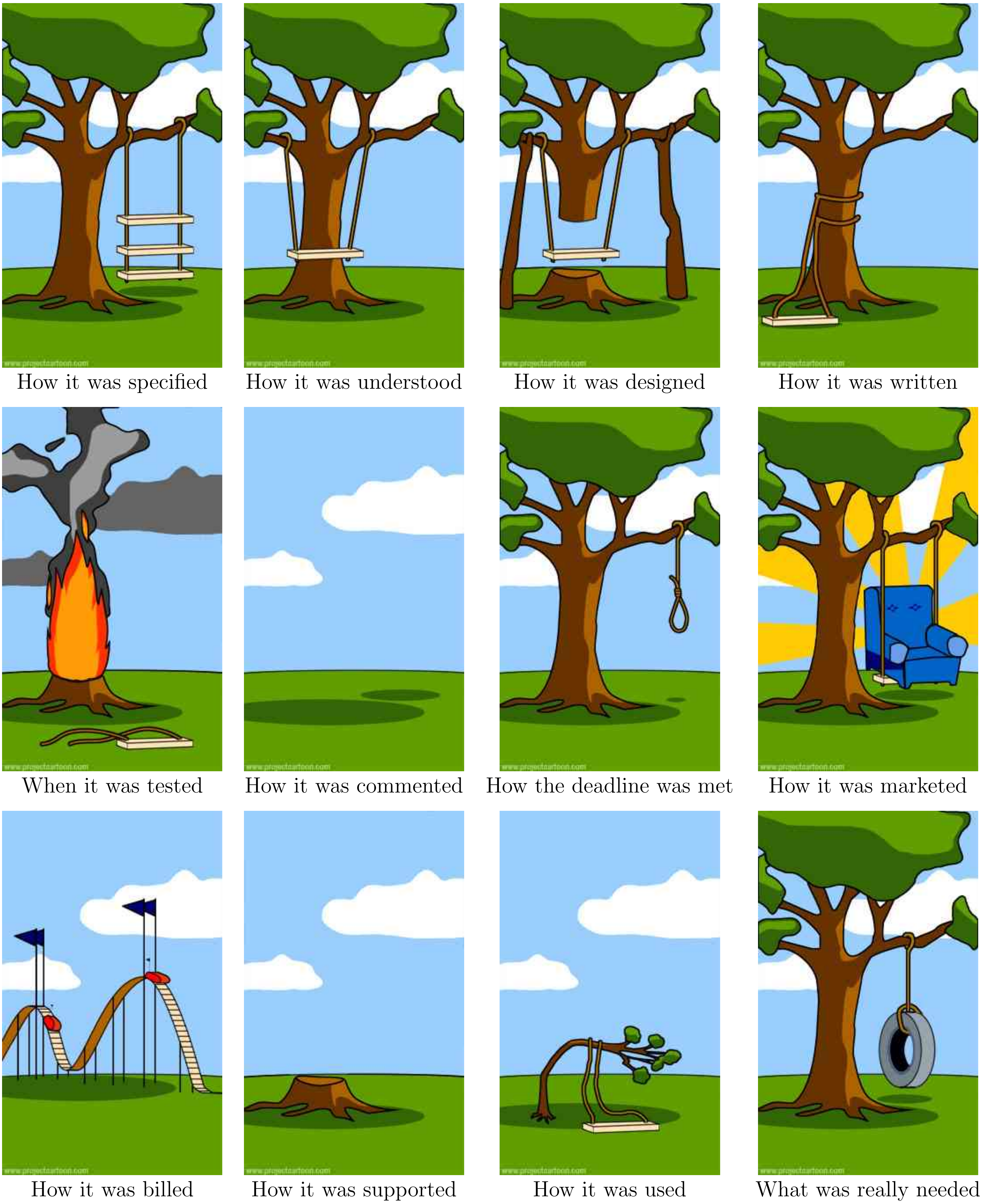


Image 3.2: Software design. (Source: www.projectcartoon.com and PA.)

- The first step in programming (in Python or any computer language) is to specify **exactly** what the program should do; specifications should be precise, accurate and complete.
- Once the problem has been specified, the programmer needs to write a sequence of commands that together solve the problem.
- Programming can be difficult, and requires a combination of technical skill, experience and creativity.
- There are many approaches that make programming easier; one is *top-down design*.
- By systematically subdividing the problem into smaller or simpler steps, and continuing to break these into even smaller steps, eventually the problem can be directly converted into lines of code.
- Using top-down design and good programming principles will:
  - make the initial programming job easier;
  - make debugging and maintaining the program easier; and
  - result in a program that is more likely to be correct.

Case Study 4: Back to BAC

- In Question 2.4.10 we modelled the time taken for blood alcohol concentrations (BACs) of male drinkers to return to 0.
- The mathematical model used the equation

$$t = 240n/m,$$

  where t is the time in hours, n is the number of standard drinks, and m is the mass of the (male) drinker in pounds.
- It is also possible to develop a computer model based on the mathematical model.

- The specifications for the program are that it must:
  - Ask the user to enter the mass (in pounds) of a male.
  - Ask the user to enter a maximum number of standard drinks.
  - Print health warning messages if the mass is less than 80 pounds, or the maximum number of drinks is more than 10.
  - Use the above mathematical model to predict the time for the BAC of the male to return to 0, after consuming each whole number of standard drinks from 1 to the maximum number.
- The following stages illustrate a top-down design process for the program. The first step is a very broad statement of what the program will do.

Program 3.1: Top-down design, BAC 1

```
1 Ask the user to input mass
2 Ask the user to input maximum drinks
3 Print health warnings if needed
4 Calculate the time for each number of drinks
```

- The second stage involves splitting Line 3 into new Lines (3 to 6) that are simpler and more like computer language commands.

Program 3.2: Top-down design, BAC 2

```
1 Ask the user to input mass
2 Ask the user to input maximum drinks
3 if the mass is < 80
4     print a health warning
5 if the maximum drinks is > 10
6     print a health warning
7 Calculate the time for each number of drinks
```

- The third stage involves splitting Line 7 into simpler steps, and also making other lines more-strongly resemble Python commands.

Program 3.3: Top-down design, BAC 3

```
1 Ask the user to input mass
2 Ask the user to input maxDrinks
3 if mass < 80
4     print a health warning
5 if maxDrinks > 10
6     print a health warning
7 loop through number of drinks n from 1 to maxDrinks
8     calculate t = 240*n/mass
9     print t
```

- The fourth stage involves splitting various lines into steps that are simpler and closer to the commands in a programming language.

Program 3.4: Top-down design, BAC 4

```
1 mass = input the mass
2 maxDrinks = input maximum drinks
3 if mass < 80
4     print a health warning
5 if maxDrinks > 10
6     print a health warning
7 numDrinks = 1
8 print an initial message
9 while numDrinks <= maxDrinks
10     calculate t = 240*numDrinks/mass
11     print t
```

- Top-down design continued for several more steps (not covered here), giving the following final version:

Program 3.5: A drinking program

```
# Model the time for BAC to return to 0 for male drinkers.
from pylab import *

# Input the mess and maximum number of drinks.
mass = eval(input("Enter the man's mass, in pounds: "))
maxDrinks = eval(input("Enter MAX drinks consumed: "))

# Do some quick health checks.
if mass < 80:
    print("This ‘man’ probably shouldn't be drinking alcohol!")
if maxDrinks > 10:
    print("Should he be drinking that much?")

# Do the time calculations for each number of drinks.
print("For a man of mass",mass,"pounds:")
numDrinks = 1
while numDrinks <= maxDrinks:
    time = numDrinks * 240 / mass
    print(numDrinks,"drinks: time is",round(time,1),"hours.")
    numDrinks = numDrinks + 1
```

***Example** 3.3.1*

If you have never seen a program before, the code may look confusing. However, you should immediately notice that:

- The program contains lines of computer commands, some of which also make some sense to a human reader – you can probably work out what some lines will do.
- Some lines look like they are messages or comments.
- Some lines are indented, and others are blank.
- Some lines look fairly mathematical.

- When a program runs, the basic rule is that each line of code is executed in turn, from the top line working down towards the bottom.
- This basic rule is modified by some commands within the program, particularly *loops* and *conditionals*.
- Lines 17–20 in the previous program form a loop.
- Lines 9–10 and 11–12 form conditionals.
- Here is the output from running the above program twice:

```
Enter the man's mass, in pounds: 160
Enter MAX drinks consumed: 5
For a man of mass 160 pounds:
1 drinks: time is 1.5 hours.
2 drinks: time is 3.0 hours.
3 drinks: time is 4.5 hours.
4 drinks: time is 6.0 hours.
5 drinks: time is 7.5 hours.


Enter the man's mass, in pounds: 70
Enter MAX drinks consumed: 11
This 'man' probably shouldn't be drinking alcohol!
Should he be drinking that much?
For a man of mass 70 pounds:
1 drinks: time is 3.4 hours.
2 drinks: time is 6.9 hours.
3 drinks: time is 10.3 hours.
4 drinks: time is 13.7 hours.
5 drinks: time is 17.1 hours.
6 drinks: time is 20.6 hours.
7 drinks: time is 24.0 hours.
8 drinks: time is 27.4 hours.
9 drinks: time is 30.9 hours.
10 drinks: time is 34.3 hours.
11 drinks: time is 37.7 hours.
```

End of Case Study 4.

3.4 Errors

- The consequences of software errors (*bugs*) can be very serious.
- Even the best and most experienced computer programmers will sometimes (even often) write programs with errors in them.

***Example** 3.4.1*

In Example 2.5.1 we noted that in 1999 the Mars Climate Orbiter (MCO) crashed into Mars as the result of a software error in relation to units. Here is an extract from the report into the accident [29]:

> ...the root cause for the loss of the MCO spacecraft was the failure to use metric units in the coding of a ground software file, "Small Forces", used in trajectory models. Specifically, thruster performance data in English units, instead of metric units, was used in the software application code titled SM_FORCES (small forces). The output from the SM_FORCES application code as required by a MSOP Project (Mars Surveyor Operations Project) Software Interface Specification (SIS) was to be in metric units of Newton-seconds (N-s). Instead, the data was reported in English units of pound-seconds (lbf-s). The Angular Momentum Desaturation (AMD) file contained the output data from the SM_FORCES software. The SIS, which was not followed, defines both the format and units of the AMD file generated by ground-based computers. Subsequent processing of the data from AMD file by the navigation software algorithm therefore, underestimated the effect on the spacecraft trajectory by a factor of 4.45, which is the required conversion factor from force in pounds to Newtons. An erroneous trajectory was computed using this incorrect data.

***Example** 3.4.2*

In 1994, Intel released the Pentium CPU chip with a software error that caused errors in mathematical calculations. It is estimated that the error cost the company around half a billion dollars.

In 2010, Toyota recalled around 500,000 hybrid-fuel cars to repair software errors that could cause the braking system to fail. There were fears that the error could lead to a "diplomatic incident" between the USA and Japan.

- There are many different types of error, including incomplete problem description, design faults in the software, unanticipated 'special cases', coding errors, logic errors and miscommunications within teams of programmers.

Testing and debugging

Most newly written programs include errors, and it is important to adopt a systematic approach to minimising the number of errors, then identifying and fixing any that occur. The process is called *testing* and *debugging*.

There are many types of programming error; some will be easy to find (like a missing bracket), some will result in error messages (like trying to divide by zero), but in many other cases the program will produce incorrect output without an error message.

To find such errors, you will need to test your program with different input values, and check the output by hand. Testing is a very important part of the overall programming process!

Avoiding errors

When writing programs, make sure that you:

- understand the specifications **before** starting;
- think about the best and most logical way to solve the problem;
- consider planning your program on paper first;
- Write your program in an organised manner, using top-down design or another systematic approach.
- Comment your program so that you (and others) know what you are trying to do;
- test your programs on a range of data;
- check some output carefully to make sure it is correct; and
- pay attention to any error messages!

Error messages are your friends!

If Python gives you error messages, make sure you use them correctly:

- do not be scared of them;
- do not ignore them: they give useful advice about what is going wrong;
- think about what they are saying;
- make full use of all of the information they give; and
- think about how you fixed similar errors in the past.

Here is an example of a Python program with an error:

Program 3.6: Errors

```
# Input a number and output that value multiplied by 2.

from pylab import *

Num = eval(input("Tell me a number: "))
Ans = Numm * 2
print("Answer = ",Ans)
```

Here is the output from running the program:

```
1 Tell me a number: 4
2 Traceback (most recent call last):
3   File "error.py", line 6, in <module>
4       Ans = Numm * 2
5   NameError: name 'Numm' is not defined
```

- The error message gives the following information:

  1. The last line of the error message (Line 5 above) identifies *what* the type of error is, in this case:

     `NameError: name 'Numm' is not defined`

2. The second line of the error message (Line 3) shows *where* the error was detected, indicating the file and line in which the error occurred.

```
File "error.py", line 6, in <module>
```

   Examine the identified line of the program and look carefully for a mistake. In this example the programmer has accidentally typed '`Numm`' instead of '`Num`' in Line 6 of the program.

- If a program contains multiple errors, Python will display the message for the first one it encounters.
- After fixing that error, a different error message may appear. *Receiving a different error message is usually a good sign*: it means that the first error is fixed, and you can move on.

Common errors

Here are some common error messages and possible causes.

- `SyntaxError` The command is not understood by Python. Perhaps:
  - your brackets are incorrect (such as `( )` instead of `[ ]`);
  - you have forgotten a bracket; or
  - your indentation is incorrect.
- `NameError` There is no variable with the given name. Perhaps:
  - you have mistyped the name of a variable; or
  - you have forgotten to set a starting value for a variable.
- `ImportError` A module to be imported does not exist. Perhaps you mistyped the name of the module to import.
- `OverflowError` The answer is too large or too small to calculate.
- `ValueError` One of the arguments you have given is not valid for the function.

3.5 Space for additional notes

Part 2: Getting Hot

Hurld headlong flaming from th' Ethereal Skie
With hideous ruine and combustion down
To bottomless perdition, there to dwell
In Adamantine Chains and penal Fire,
Who durst defie th' Omnipotent to Arms.
Nine times the Space that measures Day and Night
To mortal men, he with his horrid crew
Lay vanquisht, rowling in the fiery Gulfe
Confounded though immortal: But his doom
Reserv'd him to more wrath; for now the thought
Both of lost happiness and lasting pain
Torments him; round he throws his baleful eyes
That witness'd huge affliction and dismay
Mixt with obdurate pride and stedfast hate:
At once as far as Angels kenn he views
The dismal Situation waste and wilde,
A Dungeon horrible, on all sides round
As one great Furnace flam'd, yet from those flames
No light, but rather darkness visible
Serv'd only to discover sights of woe,
Regions of sorrow, doleful shades, where peace
And rest can never dwell, hope never comes
That comes to all; but torture without end
Still urges, and a fiery Deluge, fed
With ever-burning Sulphur unconsum'd:
Such place Eternal Justice had prepar'd
For those rebellious, here their Prison ordain'd
In utter darkness, and their portion set
As far remov'd from God and light of Heav'n
As from the Center thrice to th' utmost Pole.

Paradise Lost (c. 1677), John Milton, (1608 – 1674).

Image 3.3: *Last Judgment* (1467 – 1477), Hans Memling (c. 1430 – 1494), National Museum, Gdansk, Poland. (Source: commons.wikimedia.org).

You should be aware that, over time, the values of many natural and scientific phenomena approximately follow particular patterns. For example, they may increase or decrease, and this change can be at a constant rate, or an increasing rate, or a decreasing rate. Other phenomena have values that oscillate. When the values of a phenomenon follow a pattern, they can typically be modelled mathematically.

The next three chapters introduce some tools for mathematical modelling, specifically mathematical functions. You have encountered all of this material before, in previous study.

As is the case throughout this course, all content is presented in a particular societally relevant context. The broad context for this material is climate, and climate change, including atmospheric conditions, temperature, species diversity, seasons and environmental degradation.

There is broad scientific agreement that Earth is undergoing a period of rapid climate change, commonly called *global warming*, and that this is arising from human activities.

As we will see later, there is uncertainty in all (or most) scientific 'knowledge'. Thus, despite the strong consensus that "climate change is real", there is still popular and scientific debate about the existence, nature, causes and consequences of climate change.

From a scientific perspective, such debate is perfectly reasonable, even essential, provided it is informed, logical and based on the best available data. Unfortunately, discussions about complex issues such as climate change are typically emotive, misinformed, parochial, adversarial, alarmist or populist.

SCIE1000 is not a course on climate or climate change, so do not attempt to memorise any climate-related details. Instead, focus on the modelling tools, and *how* they can be applied to develop models in different contexts, in any area of science.

Chapter 4: **A place with atmosphere**

We are a rock revolving around a golden sun
We are a billion children rolled into one
So when I hear about the hole in the sky
Saltwater wells in my eyes.
We climb the highest mountain, we'll make the desert bloom
We're so ingenious we can walk on the moon
But when I hear of how the forests have died
Saltwater wells in my eyes.

Artist: Julian Lennon (www.youtube.com/watch?v=GzvjuMkAEEU)

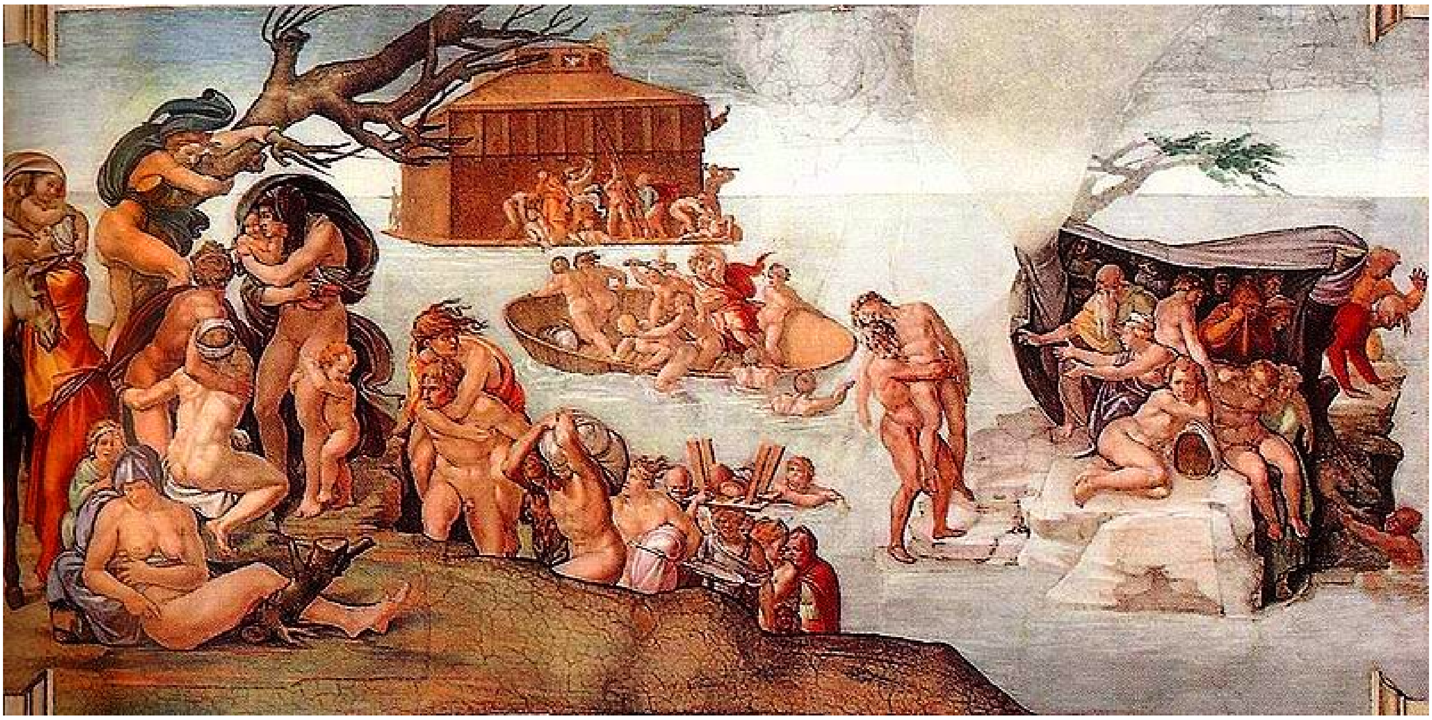


Image 4.1: *The Deluge* (1508 – 1512), Michelangelo (1475 – 1564), Sistine Chapel ceiling, Apostolic Palace, Vatican. (Source: commons.wikimedia.org)

4.1 Fully functional

Case Study 5: Atmospheric CO_2

- The broad scientific consensus is that:
  - Earth is undergoing a period of rapid climate change;
  - global temperatures are likely to rise rapidly over coming years;
  - the warming is related to increasing concentrations of carbon dioxide (CO_2) in the atmosphere; and
  - the increase in atmospheric CO_2 concentration is a result of human activity.
- A famous, long-running study has monitored atmospheric CO_2 concentrations at the Mauna Loa observatory in Hawaii since 1958.
- These data (and their graph) are called the *Keeling curve*, named after the initiator of the study.
- The website of the Scripps Institution of Oceanography (which runs the study) describes the Keeling curve as "…almost certainly the best-known icon illustrating the impact of humanity on the planet as a whole…"
- Gases in the lower atmosphere mix fairly well, so the Keeling curve is considered as representative of the atmospheric CO_2 concentration worldwide.
- The current level is around 380 parts per million by volume (ppm or ppmv).
- Other data from ice-core samples show that long-term CO_2 levels for thousands of years have remained relatively constant at 280 ppm, but started increasing in the 19th century.
- Figure 4.1 shows the Keeling curve based on data until July 2010, taken from [24, 38].

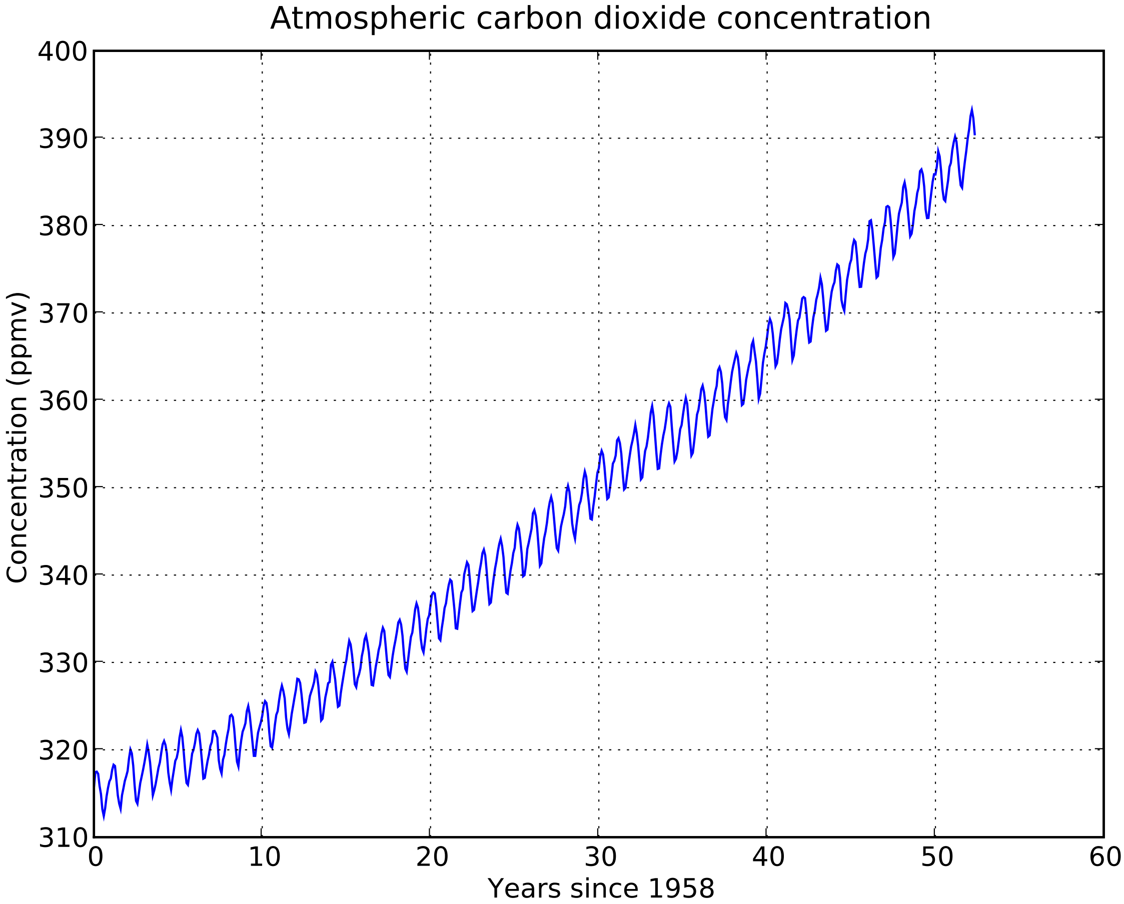


Figure 4.1: The Keeling curve.

Question 4.1.1

(a) Describe the main features of the Keeling curve graph.

(b) What physical factor(s) could be the cause of those features?

(c) How could you mathematically model the Keeling curve?

End of Case Study 5.

- Earlier we saw the importance of modelling, why most models are quantitative, and the five common ways of presenting models (using words, values, pictures, mathematics and computer programs).
- The basic mathematical tool used to describe quantitative relationships and patterns in models is the *function*.

Functions

A mathematical *function* is a rule that converts input value(s) to output value(s). If f is the name of a function, then $f(x)$ denotes the output that arises from applying f to the input value x.

- People study a range of functions, including: linear, quadratic, power, periodic, exponential, logarithmic, and combinations of these.
- All of these functions are interesting **precisely** because they model natural phenomena.
- A key skill when modelling is to recognise which type of function is most likely to best represent the observed data.
- In the next few sections we will study some phenomena and see how a range of useful mathematical functions allow us to represent and study these phenomena.
- It is not important that you memorise specific details about the particular case studies (such as the scientific name of Bicknell's thrush or the formula for wind chill).
- Instead, understand the concepts **behind** the examples, including which functions should be used to model which type of phenomena, and how to interpret mathematics in a scientific context.
- One point we will continually stress is the diversity of phenomena that can be modelled by the same (or very similar) functions.
- The first group of functions we will study are the *power functions*.

Power functions

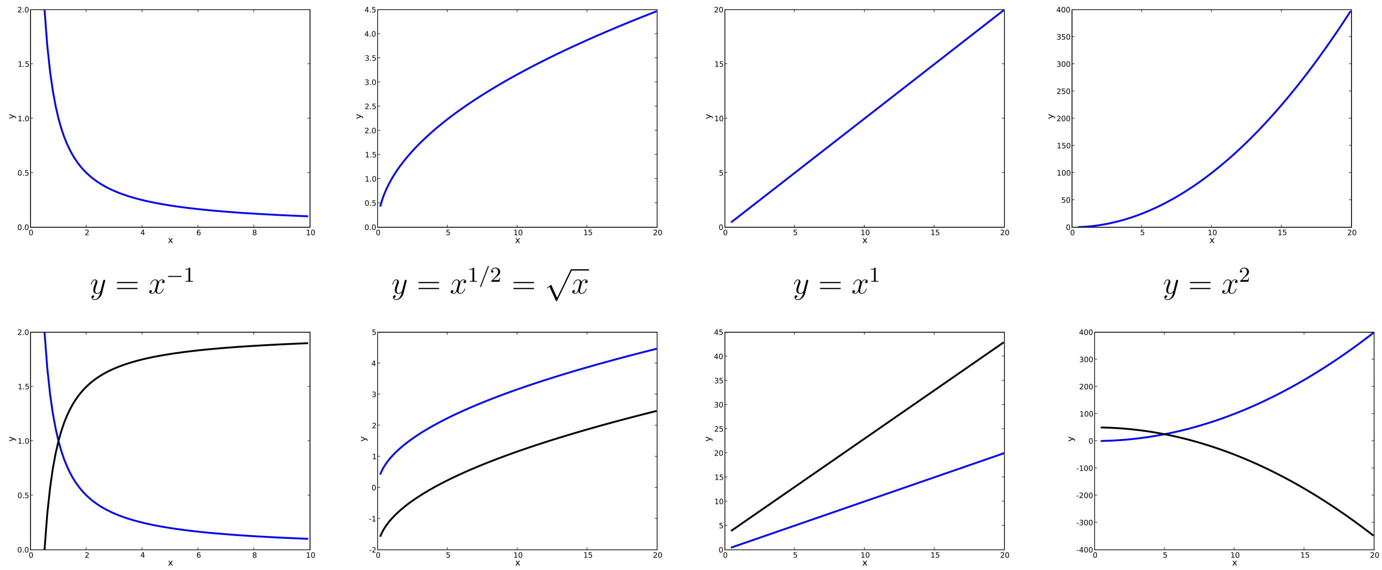
Power functions have equations $y(x) = Cx^p + k$ where C, p and k are constants. Changing the value of the various constants generates graphs with different shapes, which makes power functions useful for modelling different phenomena. For example, changing the value of:

- the power p creates graphs that increase or decrease, at different rates;
- the constant C *scales* the height of the graph vertically; and
- the constant k *shifts* the graph up or down.

Figure 4.2 illustrates how the value of the power p affects the general shape of the corresponding graph, for positive values of C and x, and Figure 4.3 shows some equations and their graphs.

Power, p	General shape of the graph
< 0	curve, decreasing less rapidly as x increases
0	horizontal line
> 0 and < 1	curve, increasing less rapidly as x increases
1	straight line
> 1	curve, increasing more rapidly as x increases

Figure 4.2: Different powers and the general shapes of the corresponding graphs.



$y = x^{-1}$ | $y = x^{1/2} = \sqrt{x}$ | $y = x^1$ | $y = x^2$

$y = x^{-1}$, $y = 2 - x^{-1}$ | $y = \sqrt{x}$, $y = \sqrt{x} - 2$ | $y = x$, $y = 2x + 3$ | $y = x^2$, $y = -x^2 + 50$

Figure 4.3: Graphs showing the shapes of some power functions.

4.2 Going straight

- Many natural phenomena are related (approximately) linearly; that is, in a *straight line. Straight lines form the basis of many techniques we will study, including linear models, calculus, Newton's method for solving equations and Euler's method for solving differential equations.*

Linear functions

Linear functions have equations $y(x) = mx + c$, where m is the *gradient and c is the y-intercept of the line.*

If (x_1, y_1) and (x_2, y_2) are two points on the line then

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Case Study 6: Temperature

Photo 4.1: Hot spring, Yellowstone Park, USA. (Source: PA.)

Question 4.2.1

Temperature conversions between the Celsius, Fahrenheit and Kelvin scales are all linear. A temperature of c degrees Celsius can be converted to equivalent temperatures K in Kelvin and F in Fahrenheit by the functions:

$$K(c) = c + 273.15 \qquad F(c) = \frac{8c}{5} + 32.$$

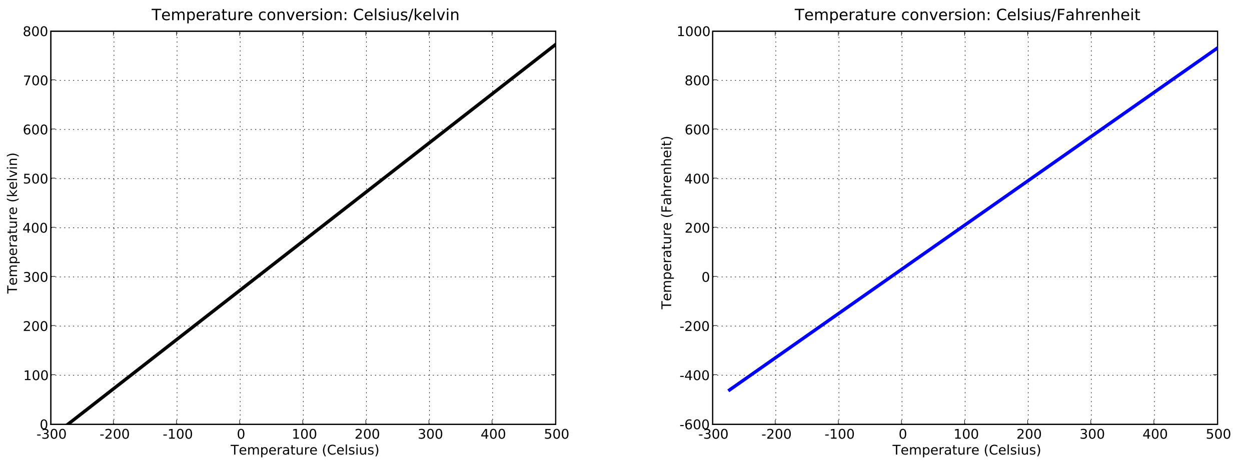


Figure 4.4: Temperature conversion graphs: $K(c)$ (left) and $F(c)$ (right).

Find the temperature(s) at which:

(a) the Fahrenheit and Celsius scales give the same reading;

(b) the Celsius and Kelvin scales give the same reading.

End of Case Study 6.

Case Study 7: Higher than a kite

Photo 4.2: Jetliner cruising at an altitude of about 10000 m. (Source: PA.)

- Scientists divide Earth's atmosphere into five primary regions: *troposphere*, *stratosphere*, *mesosphere*, *thermosphere* and *exosphere*.
- The *International Standard Atmosphere* (ISA) [22] models the atmosphere up to the base of the thermosphere using 8 layers (Layer 0 is closest to the surface of Earth).
- The ISA models various properties of the layers, including temperature, pressure and density.
- Layers in the ISA are defined as atmospheric regions in which temperature is a linear function of altitude.
- Figure 4.5 shows the relationships between temperature and altitude, as modelled by the ISA. (The ISA does not model the thermosphere; temperature data in that region are taken from other measurements.)

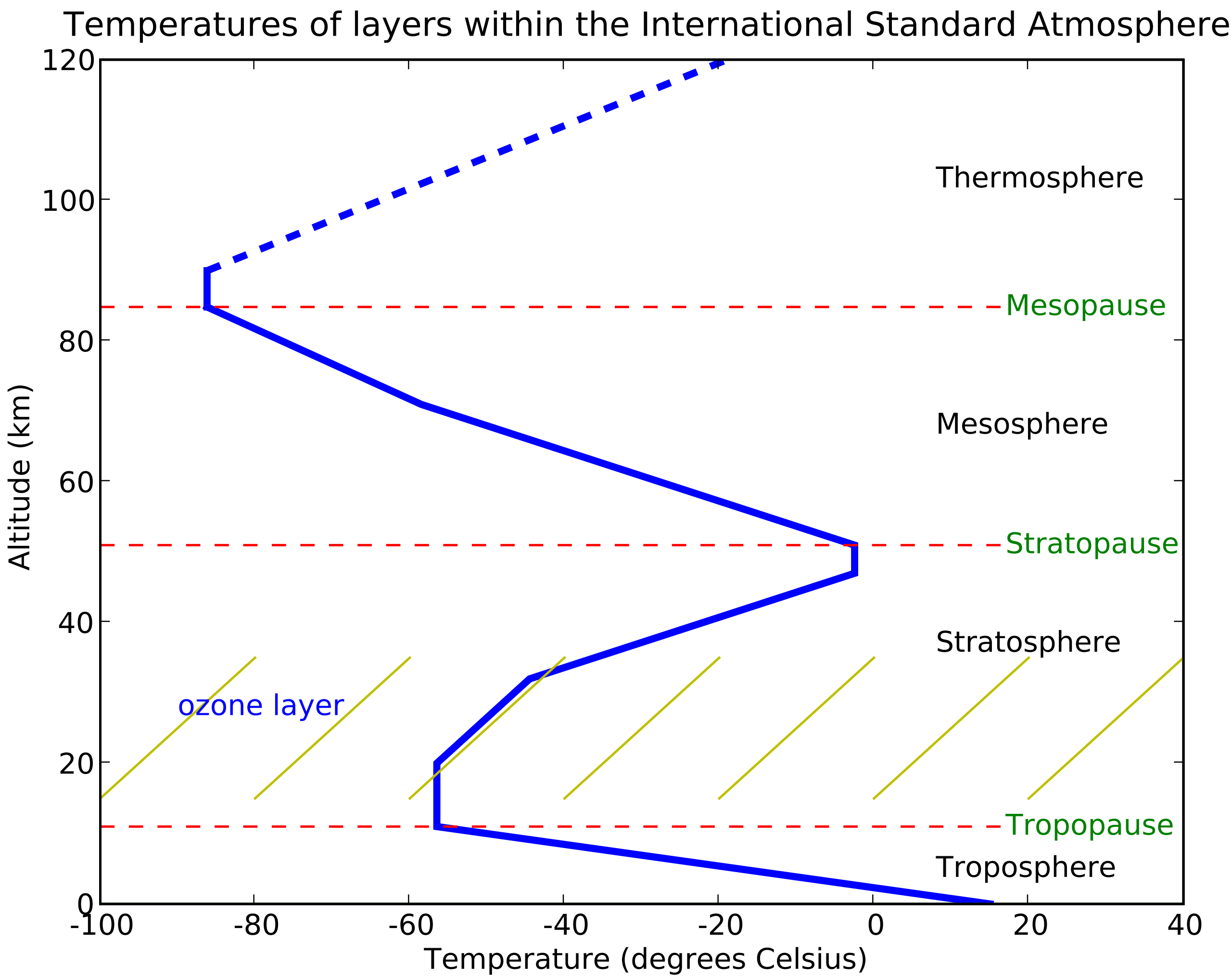


Figure 4.5: The relationships between temperature and altitude, as modelled by the ISA.

- Figure 4.6 shows various properties of the ISA at different altitudes. (The *lapse rate* is the rate at which temperature changes as altitude increases.)

Layer	Name	Height at base (km)	Lapse rate (°C/km)	Temp. at base (°C)
0	Troposphere	0.0	−6.5	+15.0
1	Tropopause	11.0	+0	−56.5
2	Stratosphere	20.0	+1.0	−56.5
3	Stratosphere	32.0	+2.8	−44.5
4	Stratopause	47.0	+0	−2.5
5	Mesosphere	51.0	−2.8	−2.5
6	Mesosphere	71.0	−2.0	−58.5
7	Mesopause	84.852	NA	−86.2

Figure 4.6: Some properties of the layers within the International Standard Atmosphere.

Question 4.2.2

Using the ISA table and/or graph:

(a) Write the troposphere temperature as a function of altitude.

(b) The Matterhorn is a mountain in the Swiss Alps, with a height of 4478 m above sea level. The summit air temperature can range from around 0 °C to −40 °C at different times of the year. Reconcile this with the temperature predicted by the ISA.

Photo 4.3: View of the Matterhorn from Italy (Source: PA.)

(continued over)

Question *4.2.2 (continued)*

(c) On a recent international flight, Peter recorded altitudes and external temperatures reported on the in-flight information screen. The data are graphed in Figure 4.7.

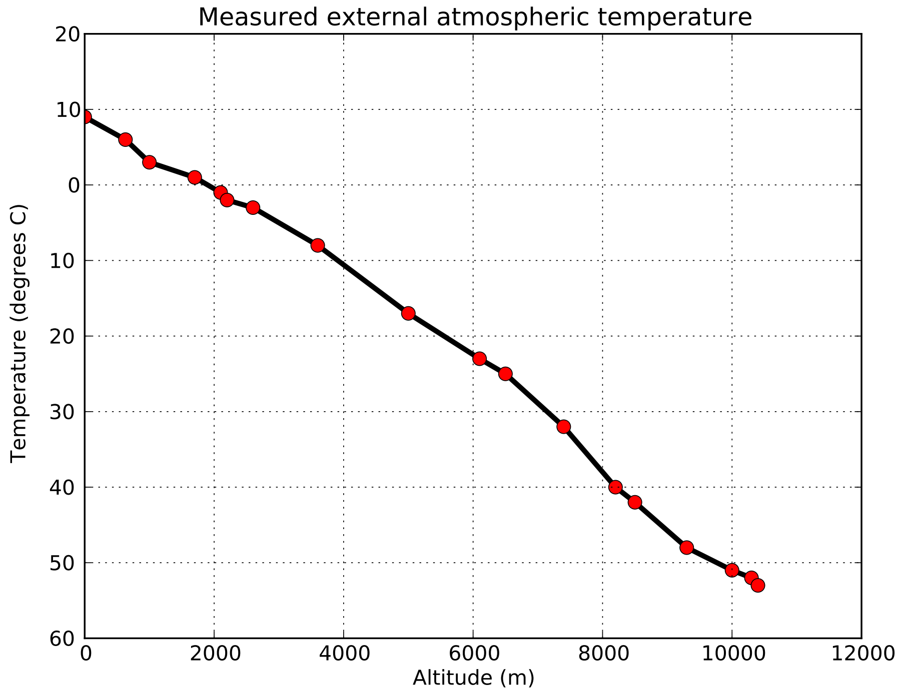


Figure 4.7: Altitudes and measured external temperatures.

Plot the function from Part (a) on the above graph and comment on the results.

(d) Write the temperature in ISA Layer 3 as a function of altitude.

End of Case Study 7.

Question *4.2.3*

First Keeling model. Figure 4.8 shows a graph of the Keeling curve.

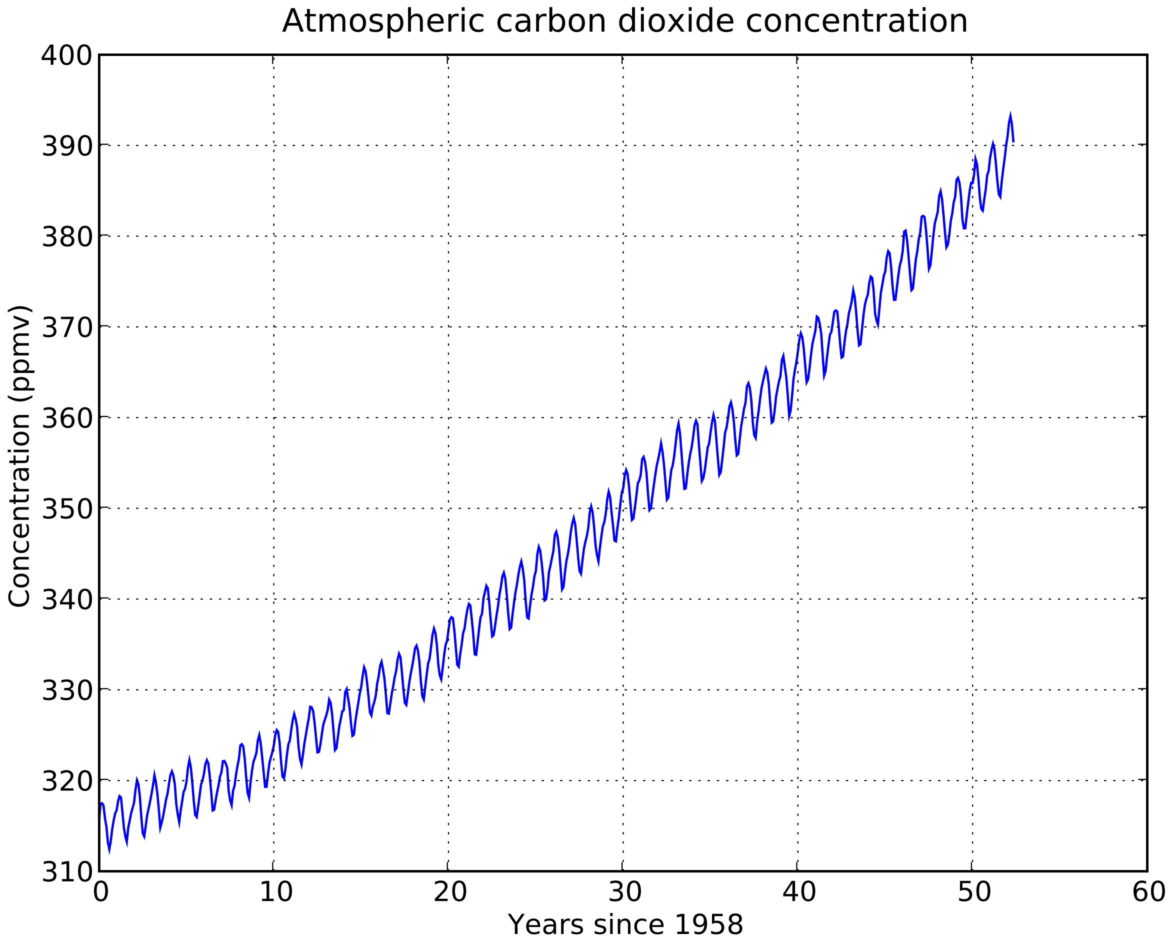


Figure 4.8: The Keeling curve.

(a) Find a rough linear model of the Keeling curve, and plot it on the graph.

(b) Discuss the effectiveness of your model.

4.3 Bend it!

- Many scientific phenomena relate in ways that are not straight lines.

Quadratics and modelling

Quadratic functions have a power of x (or t, or ...) equal to 2, with equations of the form $y(x) = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$. The graphs of quadratics are parabolas.

Quadratics are important in practical modelling, particularly when modelling over short time periods. They are the simplest functions with **optimal values, that is, maximum or minimum values.**

Case Study 8: Climate change and Bicknell's thrush

Image 4.2: Bicknell's thrush, *Catharus bicknelli.* (Source: en.wikipedia.org.)

Photo 4.4: Adirondack mountains, USA. (Source: PA.)

***Example** 4.3.1*

A paper [36] developed models for bird distributions using data from various altitudes, temperatures and locations in the north-eastern USA. The authors then used their models to predict the likely impact of rising temperatures on these distributions. Part of their study focused on Bicknell's thrush.

(continued over)

Example 4.3.1 (continued)

- Collecting data for the study involved: subdividing the study region into cells, each 30 m square; measuring the mean daily maximum temperature in July (summer) in each cell; conducting fieldwork on a representative sample of these cells to identify which contained at least one resident thrush.
- Using the data, the authors created a model for thrush distribution with respect to mean July temperatures across the breadth of their habitat.
- The study found that thrush habitats with July temperatures outside the range of 9.3 °C to 15.6 °C contained insignificant numbers of thrush.

Let t be a temperature within the range 9.3 °C – 15.6 °C. The proportion $p(t)$ of cells containing thrush is closely modelled by the quadratic function:

$$p(t) = -0.0747t^2 + 1.8693t - 10.918.$$

Question 4.3.2

The graph of $p(t) = -0.0747t^2 + 1.8693t - 10.918$ is shown in Figure 4.9.

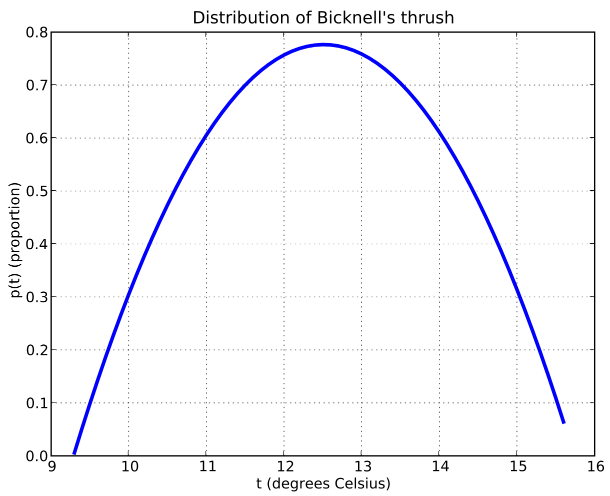


Figure 4.9: Distribution of Bicknell's thrush according to temperature.

(continued over)

***Question** 4.3.2 (continued)*

(a) What is the probability that a thrush will be found in a sample area in which $t = 11$ °C?

(b) From the graph, at what (approximate) value of t is the thrush distribution most dense, and what is the (approximate) value of $p(t)$?

(c) There is no value of t for which $p(t) = 1$. Explain what this means in terms of the thrush distribution, and give reasons why it would happen.

(d) Average temperature rises in the region over the next century are predicted to range from 2.8 °C under a low greenhouse gas emission scenario, to 5.9 °C under a high emission scenario.

(i) How would the graph in Figure 4.9 change if the average temperature rose by 2.8 °C? What if it rose by 5.9 °C? Explain your answers.

(ii) Assuming a substantial rise in average July temperatures, which key factor of concern to resident thrush would change?

(continued over)

Question 4.3.2 *(continued)*

Figure 4.10 shows the total area of existing thrush habitat, and the estimated amount of viable habitat available after predicted temperature increases under the low emission scenario and the high emission scenario.

Scenario (°C)	Habitat (hectares)
(current) +0°C	140000
+1°C	32000
+2°C	10000
+3°C	1000
+4°C	200
+5°C	75
+6°C	0

Figure 4.10: Total areas of viable habitat available to Bicknell's thrush under various climate change scenarios.

(e) What is the likely impact on the thrush population if temperatures rise by 2.8 °C or 5.9 °C?

(f) If temperature increases occur at the higher end of predictions, what kind of survival strategies might the thrush utilise?

End of Case Study 8.

Question 4.3.3

Second Keeling model. Figure 4.11 shows two plots: a graph of the function $y(t) = 0.014t^2 + 0.7t + 315$, and the Keeling curve.

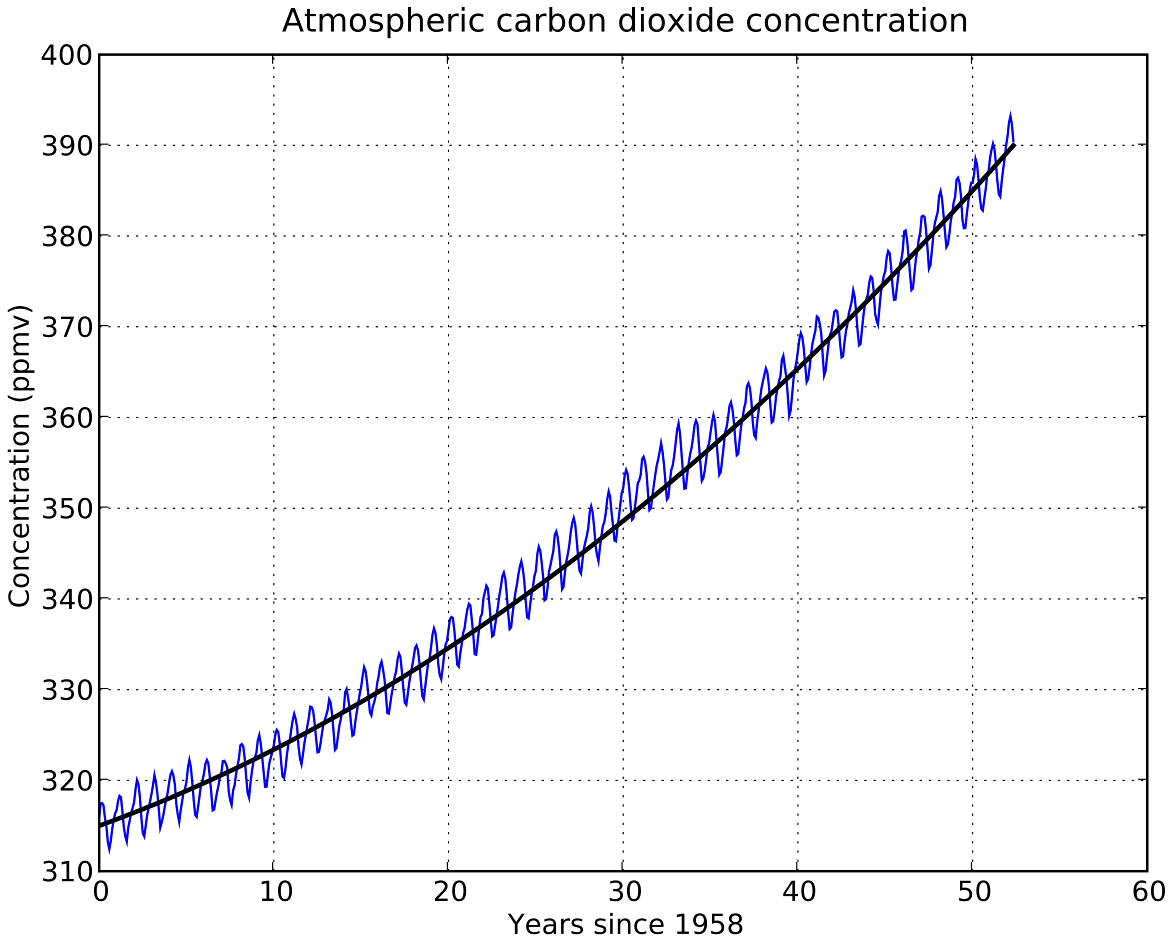


Figure 4.11: The Keeling curve and a quadratic model.

(a) Explain how each term in $y(t)$ impacts on its graph.

(b) How effectively does $y(t)$ model the Keeling curve?

4.4 (Super) powers

- Recall that linear and quadratic functions are examples of the more general group of *power functions*. Functions with different powers have graphs with different shapes, and hence can model different phenomena.

Case Study 9:

Species-area curves and biodiversity

Photo 4.5: Counting species in the field. (Source: DM.)

- Previously we discussed the abundance and distribution of a *single* species, *Bicknell's thrush*. Ecologists often study the *overall number of species* found in a region (sometimes called the *biodiversity* or *species richness*).

Photo 4.6: Scribbly gum (*Eucalyptus racemosa*). Right: Scaly-breasted Lorikeet (*Trichoglossus chlorolepidotus*). (Source: PA.)

- Rather than performing a full species count for an entire region, data from a smaller area can be extrapolated to estimate the regional species richness.

Example 4.4.1

Peter lives on 4 hectares in eastern Brisbane. He wishes to estimate the number of distinct, naturally occurring, native plant species (individuals greater than 2 m in height), that occur on his land.

He divides his land into cells of 10 m square, randomly selects 30 cells, and records the occurrence of individual plants within those cells. Figure 4.12 shows information on the previously unseen species observed in each cell or range of cells, including the scientific names of the additional species observed, and the cumulative total C of species observed so far.

Cell(s)	New species observed	C
1	*Eucalyptus racemosa*, *Acacia fimbriata*, *Banksia integrifolia*	3
2	*Eucalyptus tereticornis*, *Alphitonia excelsa*	5
3	*Acacia disparrima*	6
4	*Acacia leiocalyx*, *Lophostemon suaveolens*	8
5	–	8
6	*Glochidion sumatranum*	9
7	–	9
8	–	9
9	*Eucalyptus crebra*	10
10	–	10
11 – 15	*Banksia robur*, *Melaleuca quinquinerva*	12
16 – 20	–	12
21 – 30	*Allocasuarina littoralis*,*Angophora leiocarpa*	14

Figure 4.12: Information on additional observed species.

Species-area curves

In ecology, a *species-area curve* is a graph showing the number of distinct species observed, as a function of the size of the area surveyed.

Example 4.4.2

Figure 4.13 is a species-area curve summarising the data in Figure 4.12:

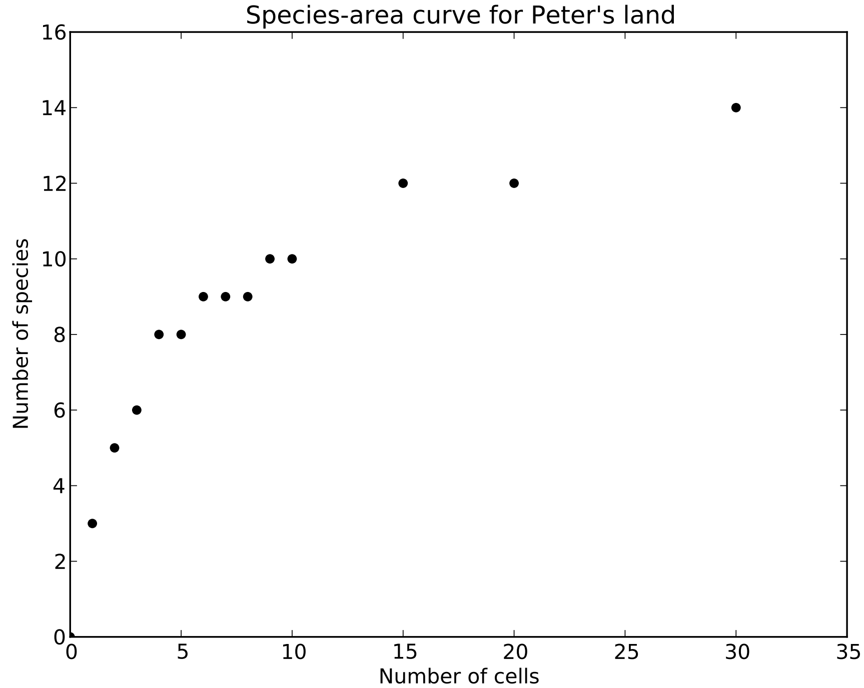


Figure 4.13: The number of distinct tree species recorded on Peter's land.

- The graph has a shape that is typical of many species-area curves: the number of distinct species initially rises rapidly as the area increases, but then rises less rapidly as the area becomes larger.

Equations for species-area curves

Species-area curves are most often mathematically modelled using power functions, with power p between 0 and 1 (typically, p is between 0.2 and 0.5).

Their general form is $S(a) = Ca^p$, where S is the number of species occurring as a function of the area a, and C and p are constants depending on the geographical location, resource availability and biological diversity of that environment.

Question 4.4.3

With respect to a species-area curve $S = Ca^p$ (with p between 0 and 1):

(a) Give some physical reasons or the general shape of species-area curves.

(b) How might this impact on field sampling techniques?

(c) Describe some physical features that would make the values of C and p smaller or larger.

Example 4.4.4

Figure 4.14 shows the graph of $f(a) = 5a^{0.3}$ and the species data from Figure 4.12, where a is the number of 10 m square cells on Peter's land.

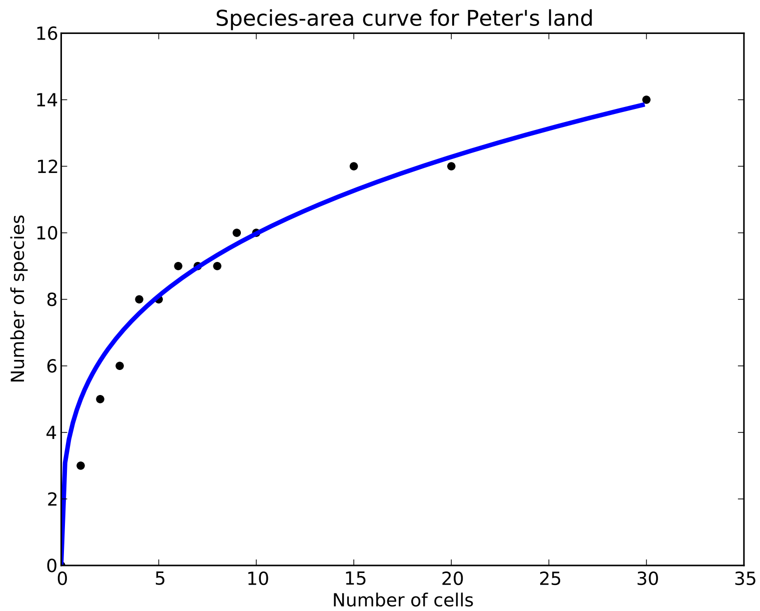


Figure 4.14: Modelling the species data from Peter's land.

Question 4.4.5

Assume that this question refers to native, naturally occurring plants more than 2 m high, growing on land ecologically similar to Peter's.

(a) Estimate the species richness on Peter's 4 hectare (40000 m^2) property.

(b) A typical conservation goal is to establish parks that preserve 10% of representative land area. What relative species richness would be represented in such a park in the area in which Peter lives?

(c) Many people believe that the figure in Part (b) is too low. If the goal is to retain 75% of species, what proportion of land should be preserved?

- Species-area curves have been used to predict impacts of climate change.

Example 4.4.6

The paper [44] uses three models based on species-area curves to predict possible loss of species richness in the event of climate change. One model for the proportion E of species predicted to become extinct is:

$$E = 1 - \left(\frac{\sum A_n}{\sum A_o} \right)^{0.25}$$

where the summations occur over all of the species in a study, A_n is the predicted new habitat size of a species after climate change, and A_o is the original habitat size of that species.

Amongst other results, the paper [44] deduces the following possible extinction rates within various taxa of organisms in Queensland:

- mammals: 10% for a conservative climate change scenario, and 50% for a maximal scenario
- birds: between 7% and 49%
- frogs: between 8% and 38%
- reptiles: between 7% and 43%

The results in this paper were very widely reported, and have attracted substantial scientific debate.

Question 4.4.7

With reference to the model used in [44] and given in Example 4.4.6:

(a) Justify the model.

(continued over)

***Question** 4.4.7 (continued)*

(b) Criticise the model.

(c) Earlier, Figure 4.10 showed the predicted areas of habitat available to Bicknell's thrush under various climate change scenarios; the current habitat is 140000 hectares, which is predicted to reduce to 10000 hectares under a medium CO_2 emissions scenario, and to 75 hectares under a high emissions scenario.

Consider a group of species that (for simplicity) exactly mirrors the distribution and climatic requirements of Bicknell's thrush. Predict the likely loss of species richness within this group in the event of medium and high CO_2 emissions scenarios.

End of Case Study 9.

Case Study 10:
Ban the tan, man

- Earlier, we saw that in the International Standard Atmosphere, temperature in the lowest atmospheric layer (the Troposphere) decreases as altitude increases, but temperature in the next atmospheric layer (the Stratosphere) increases as the altitude increases from 20 km to 50 km.
- The rise in temperature is due to interations between the ozone layer and ultraviolet (UV) light.
- Ultraviolet light is electromagnetic radiation with wavelengths shorter than that of visible light, and can be divided into: UV-A with wavelength between 315 and 400 nm; UV-B with wavelength 280 – 315 nm; and UV-C with wavelength 100–280 nm.
- The following sequence of chemical reactions occurs in the ozone layer:

$$\begin{aligned} O_2\ (g) &\xrightarrow{\text{UV-C light}} 2O\ (g) \\ O_2\ (g) + O\ (g) &\longrightarrow O_3\ (g) \\ O_3\ (g) &\xrightarrow{\text{UV-B light}} O_2\ (g) + O\ (g) \end{aligned}$$

- In the first reaction, high-energy UV-C light is absorbed. The second reaction is exothermic, so the net result of the first two reactions is that light energy is converted to heat, and oxygen is converted into ozone.
- In the third reaction, UV-B light is absorbed; this reaction is also exothermic, again converting light energy into heat.
- Collectively, these reactions are called the *ozone-oxygen* cycle. This cycle is extremely important to life on Earth.
- UV-C light is high-energy, and would be very damaging to life. It is completely absorbed in the atmosphere.

- Most UV-B light is absorbed in the atmosphere; only around 1 part in 350 million reaches the surface of Earth. Almost all UV-A light reaches the surface of Earth.
- Exposure to UV light causes skin *tanning*.
- Exposure to UV-B light can cause sunburn, eye cataracts, visible ageing, genetic mutations in cells, and skin cancer.
- To prevent damage from the sun, health authorities recommend applying *sunscreens* to the skin.
- The effectiveness of sunscreens at preventing UV light from reaching the skin is measured by their *Sun Protection Factor*, SPF.
- Effectively, when a product with SPF n is correctly applied to the skin, it blocks a fraction of $(n-1)/n$ of the usual amount of UV-B light.

***Question** 4.4.8*

Assume that a product with SPF n is applied correctly.

(a) Write a function for the proportion of UV-B light that is **not** blocked.

(b) Draw a rough sketch of the graph from Part (a).

(continued over)

Question *4.4.8 (continued)*

(c) A **very rough** rule of thumb is: sunscreen with SPF n increases the time it takes you to burn by a multiple of n. Justify this.

(Do not rely on this rule of thumb; there are many other factors, including changing sun intensity and the sunscreen chemicals being removed from the skin, for example by water.)

- It is well-known that there has been a substantial depletion of atmospheric ozone over the last few decades, manifested by an ongoing gradual decline in total atmospheric ozone volume, and also by the *ozone hole* over Antarctica.
- It is well-accepted that anthropogenic activity is responsible for this, in particular the release of *ozone depleting* substances into the atmosphere. These include *chlorofluorocarbons*, which were previously used as refrigerants and aerosol propellants.
- The *Montreal Protocol*, adopted in 1989, is an international agreement on phasing out the use of CFCs.
- This protocol has been ratified by almost 200 states, and represents one of the most significant international agreements ever.
- Scientists believe that the ozone layer will recover by the year 2050.

End of Case Study 10.

- Previous examples have shown how some simple mathematical functions are used to model various phenomena, and how to interpret these models.
- Next we build on the functions we have studied, by combining multiple physical factors into models.
- Rather than a single independent variable (like time t or area a), the next example considers how ambient temperature and wind speed combine to change the apparent temperature that we perceive.

Case Study 11: Wind chill

Photo 4.7: Blizzard, West Yellowstone, USA. (Source: PA.)

- Windy days can feel much colder than calm days, even if ambient air temperatures are the same on both days.
- Particularly on cold days, the *apparent* temperature to the human body drops as the wind speed increases.
- This effect is commonly called wind chill.

- Because wind chill can cause major discomfort, and in cold climates can lead to serious injuries such as frostbite or even death, it is important to measure, model and predict the severity of wind chill.

Question *4.4.9*

Derive an equation that models wind chill. (Hint: start by deciding which factors are important, whether they increase or decrease the apparent temperature, whether their effect is linear, and how they interact.)

- It is possible to measure wind chill in a number of ways. In 2001, the US National Weather Service developed the model that is currently most widely accepted model.
- Researchers exposed volunteers to various low temperatures and high wind speeds in a wind tunnel, recording their perceptions of temperatures, along with measurements of the physiological impact of wind chill on their faces.
- The researchers then formulated an equation that modelled the perceived wind chill temperature as a function of the ambient air temperature and the wind speed (for speeds of at least 5 km/h).

Question *4.4.10*

Let t be the ambient air temperature in °C and v be the wind speed in km/h. Then the wind chill temperature W perceived by the human body in °C is given by the equation:

Example *4.4.11*

On a cold Brisbane bike ride, the ambient temperature is 2 °C and the effective wind speed is 30 km/h. Thus,

$$W \approx 13.12 + 1.24 - 11.37 \times 1.723 + 0.79 \times 1.723 \approx -3.85,$$

so the perceived temperature is about -3.85 °C.

We can now use develop a computer model of wind chill calculations.

Program specifications: Write a program that allows the user to input wind speed in km/h and air temperature in °C, and then calculates the apparent wind chill temperature.

Program 4.1: Wind chill

```
# A program to calculate apparent wind-chill temperatures.
from pylab import *

airT = eval(input("Enter air temp. in degrees Celsius: "))
windS = eval(input("Enter wind speed in km/h: "))
x = pow(windS,0.16)
windC = 13.112 + 0.6215 * airT - 11.37 * x + 0.3965 * airT * x
rt = round(windC,1)

print("An air temp. of ",airT," Celsius and wind speed of")
print (windS,"km/h gives a wind chill of",rt," Celsius.")
```

Here is the output from running the above program twice:

```
Enter air temp. in degrees Celsius: -19
Enter wind speed in km/h: 19
An air temp. of  -19  Celsius and wind speed of
19 km/h gives a wind chill of -29.0  Celsius.

Enter air temp. in degrees Celsius: -36
Enter wind speed in km/h: 135
An air temp. of  -36  Celsius and wind speed of
135 km/h gives a wind chill of -65.5  Celsius.
```

Photo 4.8: Mont Blanc. (Source: PA.)

Example 4.4.12

Figure 4.15 shows average maximum temperatures and wind speeds on the summit of Mount Everest for the years 2002 – 2004 (see [13]), and Figure 4.16 shows the corresponding wind chill temperatures.

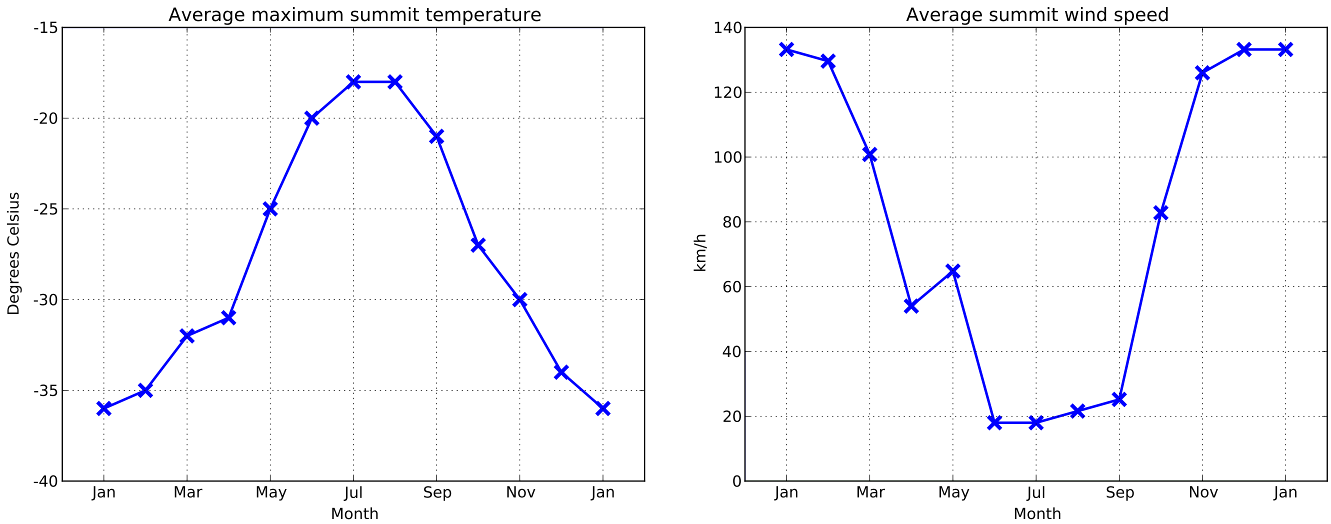


Figure 4.15: Wind speeds and temperatures on the summit of Mount Everest.

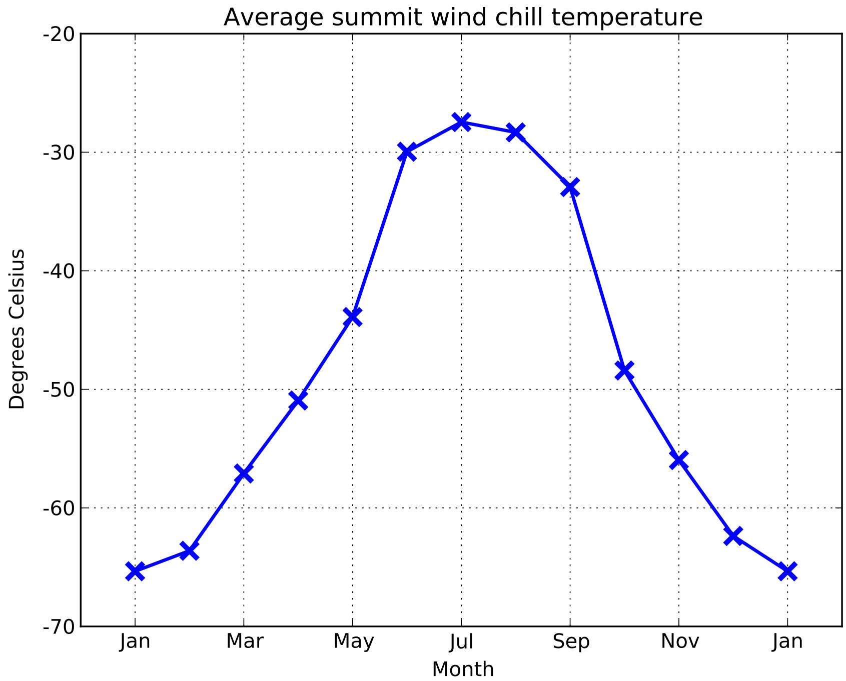


Figure 4.16: Wind chill temperatures on the summit of Mount Everest

There are two very short annual "windows" during which conditions are typically most suitable for ascending to the summit of Mount Everest: May 20 to June 6, and Oct 1 to Oct 20.

Question *4.4.13*

Making wind chill information accessible and comprehensible can be a matter of life and death. How effective is each of the five common ways of presenting quantitative models (words, values, pictures, equations and computer programs) at making wind chill information widely available?

(a) Words:

(b) Values:

(c) Pictures (such as graphs):

(d) Equation:

(e) Computer program:

In practice, the most common way to present wind chill information is with a table of values, often with colour coding to show the risk of developing frostbite. (Frostbite is a medical condition in which intense cold causes tissues to freeze and eventually die.) An example of such a table is shown in Figure 4.17.

		Air temperature (degrees Celsius)												
		10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
Wind Speed (km/h)	5	10	4	-2	-7	-13	-19	-24	-30	-36	-41	-47	-53	-58
	10	9	3	-3	-9	-15	-21	-27	-33	-39	-45	-51	-57	-63
	15	8	2	-4	-11	-17	-23	-29	-35	-41	-48	-54	-60	-66
	20	7	1	-5	-12	-18	-24	-30	-37	-43	-49	-56	-62	-68
	25	7	1	-6	-12	-19	-25	-32	-38	-44	-51	-57	-64	-70
	30	7	0	-6	-13	-20	-26	-33	-39	-46	-52	-59	-65	-72
	35	6	0	-7	-14	-20	-27	-33	-40	-47	-53	-60	-66	-73
	40	6	-1	-7	-14	-21	-27	-34	-41	-48	-54	-61	-68	-74
	45	6	-1	-8	-15	-21	-28	-35	-42	-48	-55	-62	-69	-75
	50	5	-1	-8	-15	-22	-29	-35	-42	-49	-56	-63	-69	-76
	55	5	-2	-8	-15	-22	-29	-36	-43	-50	-57	-63	-70	-77
	60	5	-2	-9	-16	-23	-30	-36	-43	-50	-57	-64	-71	-78
	65	5	-2	-9	-16	-23	-30	-37	-44	-51	-58	-65	-72	-79
	70	5	-2	-9	-16	-23	-30	-37	-44	-51	-58	-65	-73	-80

Risk of developing frostbite:	
Low:	< 5% chance of developing frostbite
Increasing:	5% - 95% chance of developing frostbite in 10 to 30 mins.
High:	> 95% chance of developing frostbite in 5 to 10 mins.
Very high:	> 95% chance of developing frostbite in 2 to 5 mins.
Extreme:	> 95% chance of developing frostbite in 2 mins.

Figure 4.17: Wind chill temperatures at various ambient temperatures and wind speeds, colour-coded with frostbite risk factors.

End of Case Study 11.

Question 4.4.14

Third Keeling model. Figure 4.18 shows a graph of the Keeling curve.

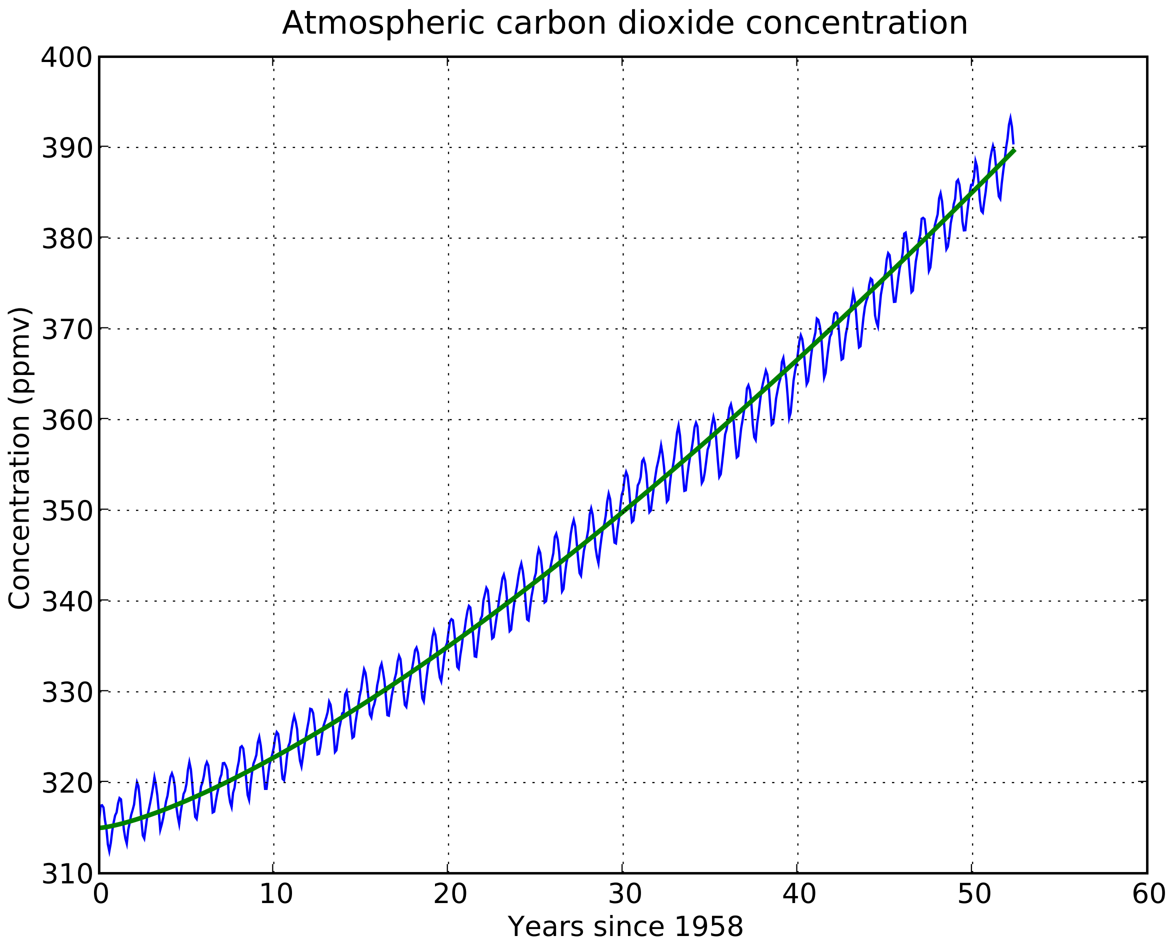


Figure 4.18: The Keeling curve and a power-function model.

(a) Figure 4.18 also includes a plot of $y(t) = 1/3 \times t^{1.367} + 315$, for t between 0 and 52. Explain how each term in $y(t)$ impacts on the graph.

(b) How effectively does $y(t)$ model the Keeling curve?

4.5 Space for additional notes

Chapter 5: **Give us a wave!**

Goodbye papa it's hard to die
When all the birds are singing in the sky
Now that the spring is in the air
Little children everywhere
When you see them I'll be there.
We had joy we had fun,
We had seasons in the sun.
But the wine and the songs
like the seasons have all gone.

Artist: Terry Jacks

(www.youtube.com/watch?v=iA6BqS9FlQ0)

Image 5.1: *God creating the Heavens and Earth* (*The separation of Light and Darkness* (left), *The creation of the Sun, Moon and Earth* (centre), *The separation of Land and Water* (right)) (1508 – 1512), Michelangelo (1475 – 1564), Sistine Chapel ceiling, Apostolic Palace, Vatican. (Source: en.wikipedia.org)

5.1 Waves, cycles and periodic functions

- Many phenomena in Science and nature *repeat* or *cycle*. These include: many aspects of weather and climate; ocean waves and tides; physiological processes, such as breathing and hormone levels; sound waves; and the voltages and currents in alternating current electricity.
- Consider the four graphs in Figure 5.1, each of which shows climate-related data for Brisbane over a period of one year. If the graphs were extended over subsequent years, then an approximate cycling pattern would be observed.

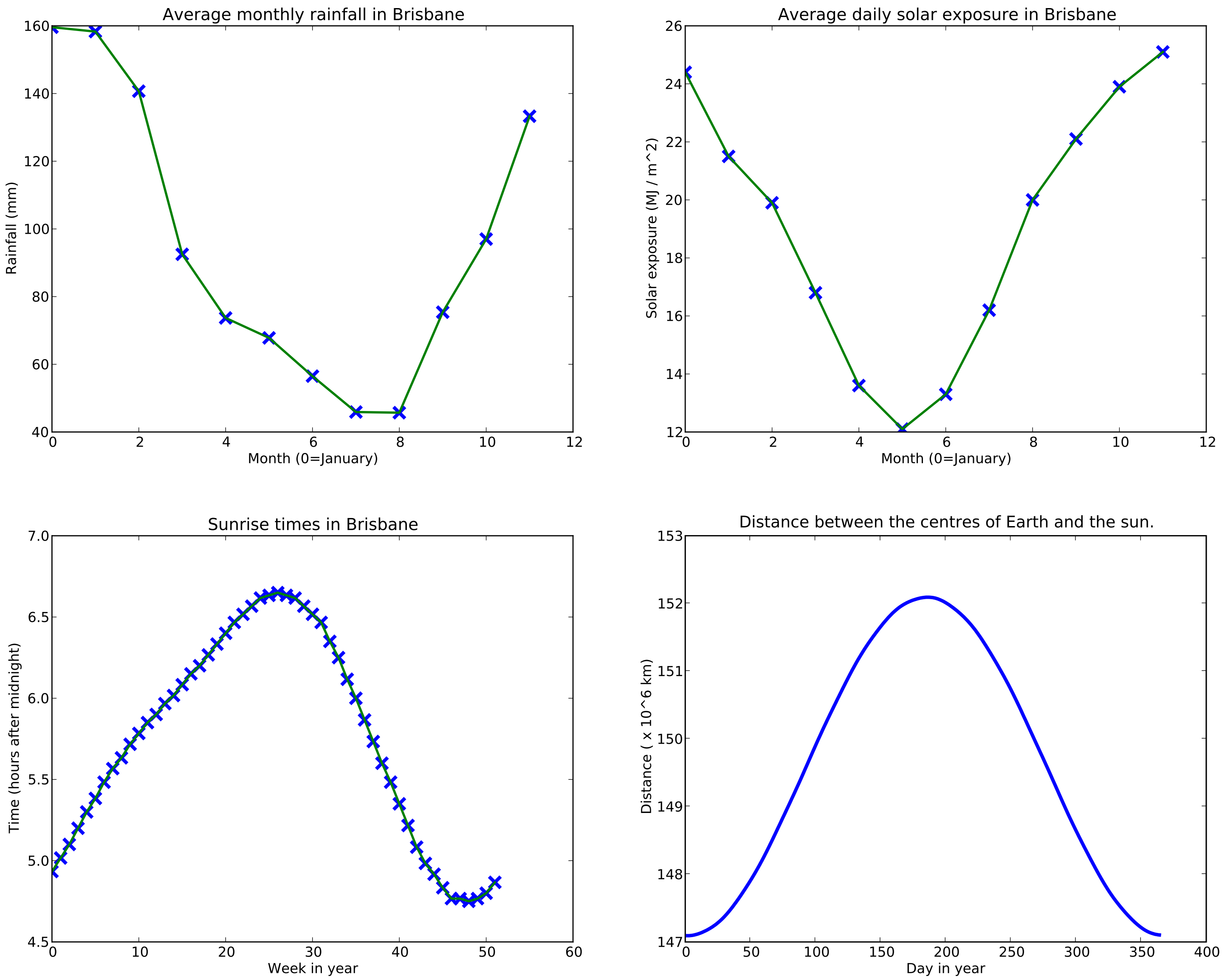


Figure 5.1: Four climate-related graphs. Top left: average monthly rainfall in Brisbane. Top right: average daily solar exposure in Brisbane. Bottom left: weekly sunrise times in Brisbane. Bottom right: daily distances between the centres of Earth and the sun.

Waves

Graphs of cyclic phenomena are called *waves*. Properties of waves include:

- **peaks and troughs** – highest and lowest values on the wave;
- **equilibrium value** – value around which the wave is centred.
- **wavelength** – distance of one **cycle**, from one peak to the next;
- **amplitude** – largest deviation from the equilibrium during a cycle;
- **phase shift** – partial horizontal shift of the wave;
- **period** – time taken for one complete cycle; and
- **frequency** – the rate at which peaks pass a given point, equal to the reciprocal of the period, measured in cycles per second, **hertz** or **hz**.

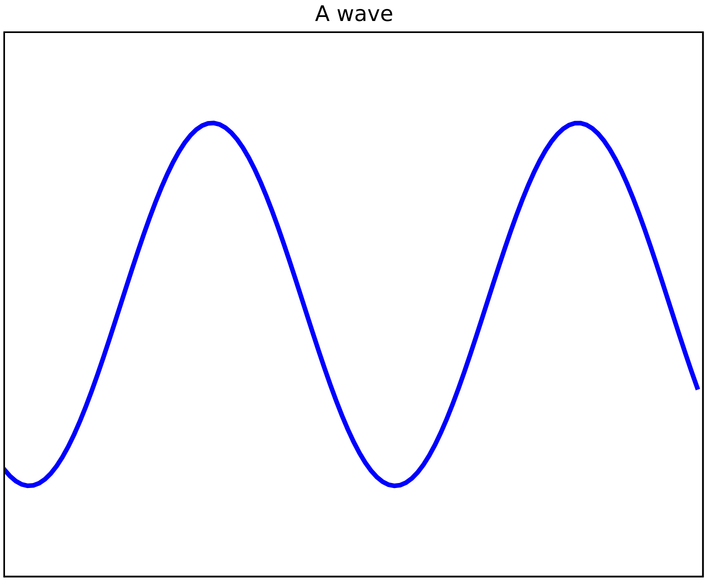


Figure 5.2: The graph of a wave.

- To represent waves accurately we require a new type of function. It doesn't matter **which** functions are used, provided they look like waves. The most common choices are the *trigonometric functions* sin and cos.
- These functions are defined in the context of geometry and angles. However, **do not** think of them in a geometric context when modelling scientific phenomena. They are useful precisely because they cycle, so can be used to model cyclic phenomena. This has nothing directly to do with angles!

- In SCIE1000 we will always use the function sin (we could have used cos).

The periodic function sin

Figure 5.3 shows the graph of $y = \sin x$ for x between -2π and 2π; the graph shows two cycles of a *sine wave.*

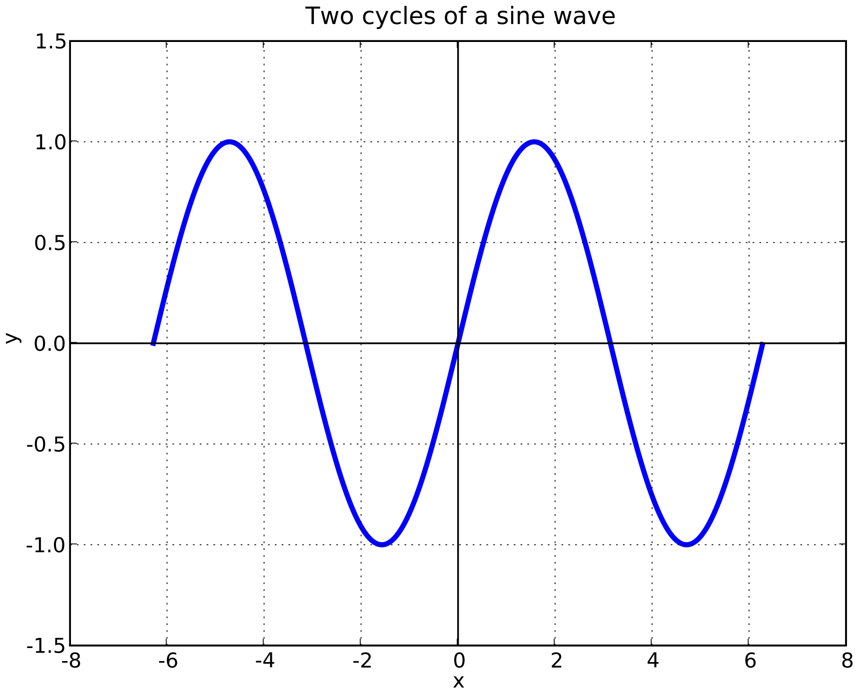


Figure 5.3: Two cycles of a sine wave.

- Because of the way in which it is defined, the function $\sin x$ has a period of 2π, an amplitude of 1, an equilibrium value of 0, and the function equals 0 when time equals 0.
- Of course, cyclic phenomena in nature typically have different properties. Varying the values of the "constants" within a sin function alters the properties of the cyclic model.

Question *5.1.1*

Four graphs of $y = \sin t$ are shown below. For each given scenario, write an equation for a sin function that models the stated property, then sketch the new function on the graph provided.

(continued over)

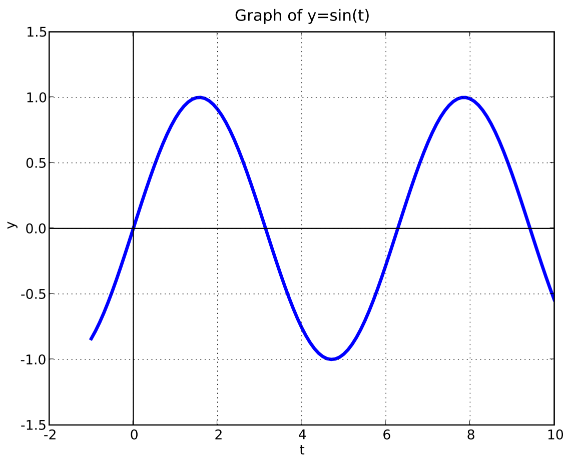
Question *5.1.1 (continued)*

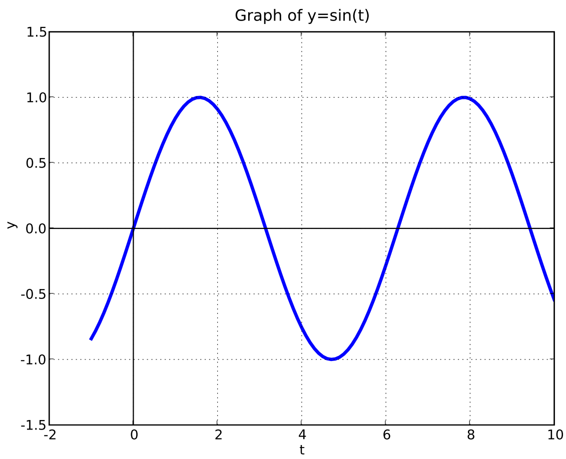
(a) Centred around $y = 0.5$.

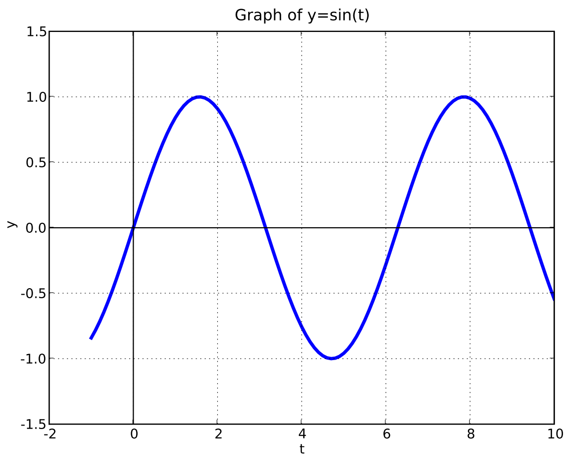
(b) Amplitude of 0.5.

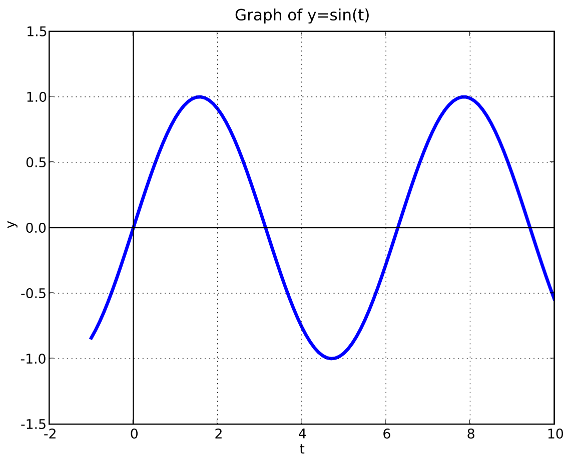
(c) A period of 5.

(d) A phase shift of one half of a cycle.









Case Study 12: Heavy breathing

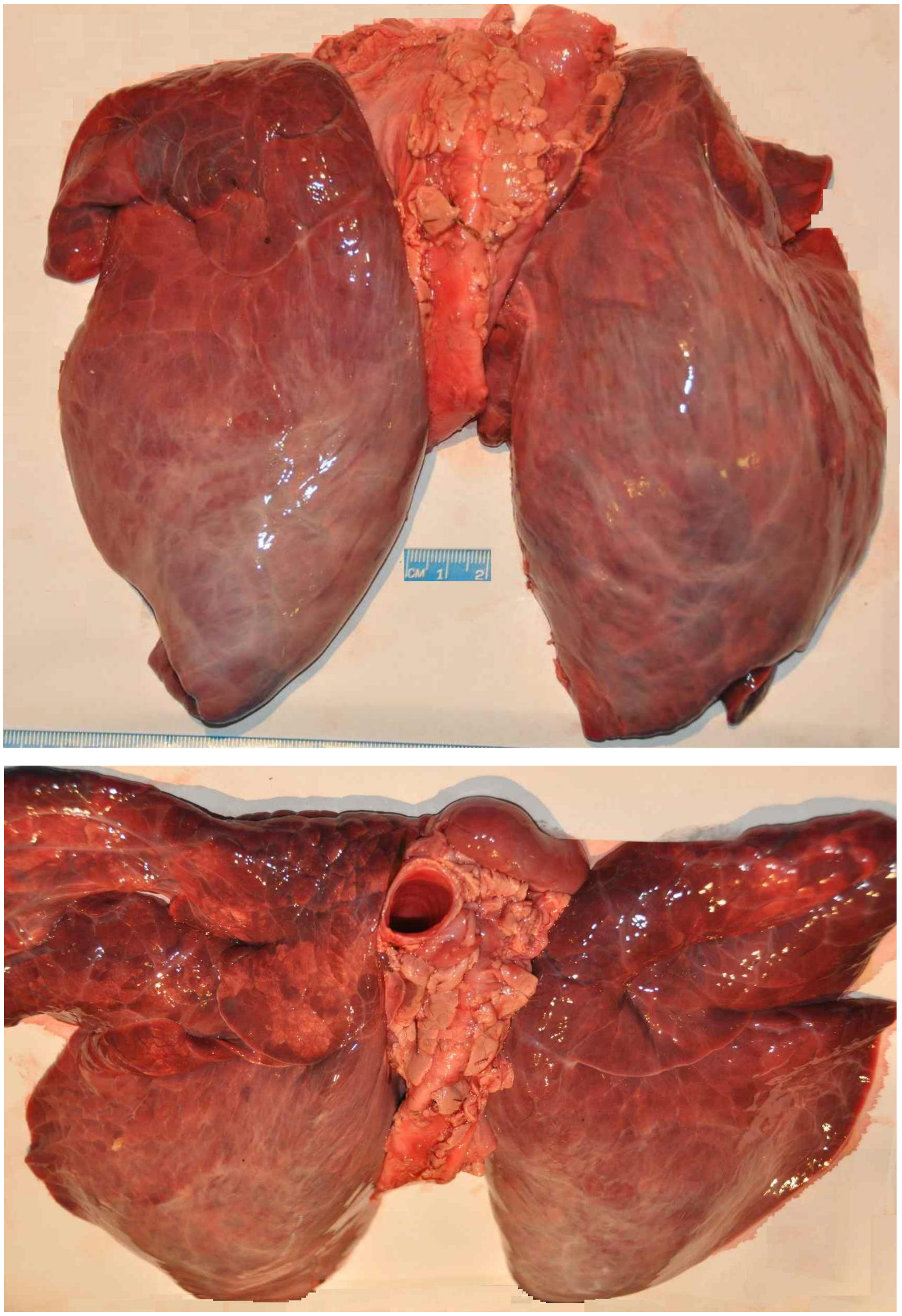


Photo 5.1: Calf lungs. (Source: PA.)

- All lungs have a maximum capacity, determined by factors such as the size, gender and level of physical activity of the individual. The total lung capacity of an adult human male is typically around 6 L.
- Normal breathing involves rhythmic inhalation and exhalation of air. The **tidal volume** is the total volume of air breathed in and out with normal

breathing. After each exhalation the lung retains a volume of air, called the **functional residual capacity**.

- The volume and rate of air movement into and out of the lungs can be measured using a *spirometer* and graphed in a *spirogram*. (One common type of spirometer uses the Hagen-Poiseuille equation to measure air flow rates.) This information can be used to diagnose possible respiratory impairment.

Question *5.1.2*

(a) Estimate the functional residual capacity, tidal volume and period between breaths for a resting adult human, and sketch a rough graph of lung capacity (that is, the volume of air in the lung) over time.

(b) Write a function using sin to model the lung capacity in Part (a).

(c) How would the function change after intense physical activity?

(continued over)

Question *5.1.2 (continued)*

(d) Hyperventilation is characterised by rapid, deep inhalations and exhalations. How would the function change during hyperventilation compared to normal breathing?

(e) Smoking and air pollution causes inflammation in the lungs, gradually destroying the lung tissue and leading to *emphysema*, a type of Chronic Obstructive Pulmonary Disease (COPD). The reduction of lung surface area decreases the ability to exchange carbon dioxide and oxygen.

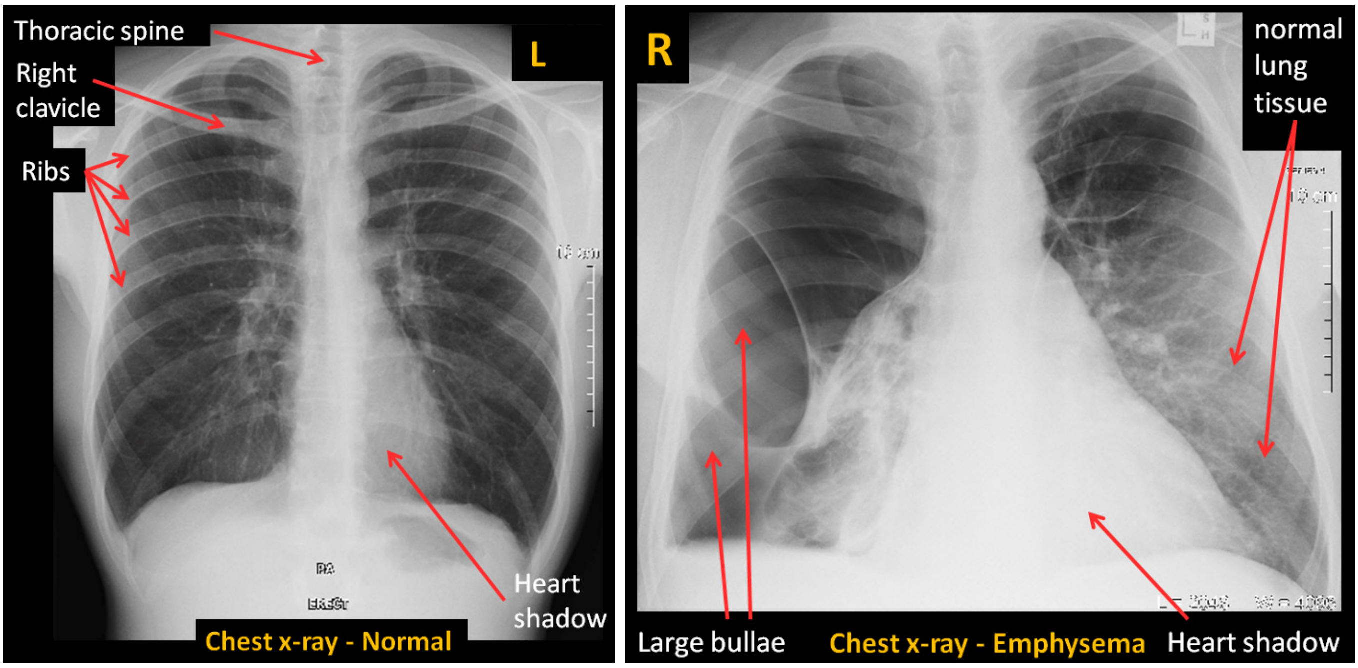


Photo 5.2: Left: x-ray of an adult male chest displaying normal lung tissue architecture and normal heart shadow. Right: x-ray of a chest showing large emphysematous bullae within the right lung. (Source: Qld Health and DM.)

(continued over)

Question 5.1.2 *(continued)*

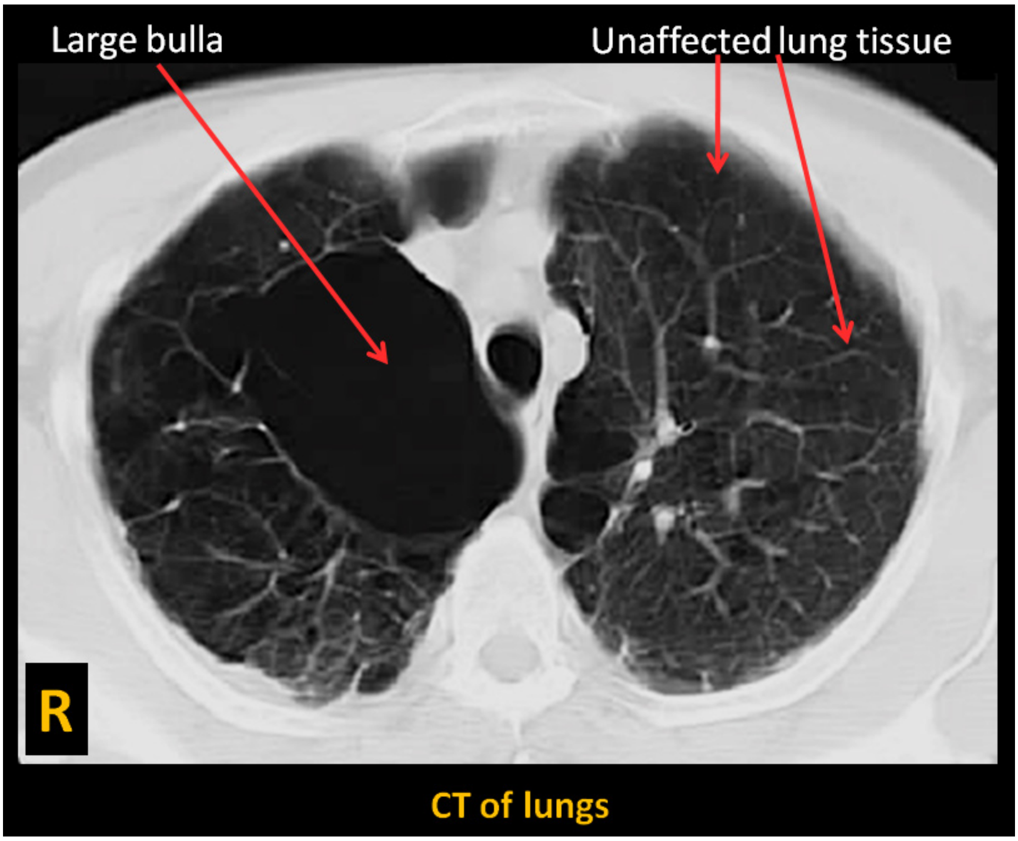


Photo 5.3: Axial CT showing one large, and multiple small, bullae of the alveolar air spaces in the right lung. (Source: Qld Health and DM.)

How would the function change for an individual with emphysema?

End of Case Study 12.

5.2 Days, seasons, cycles

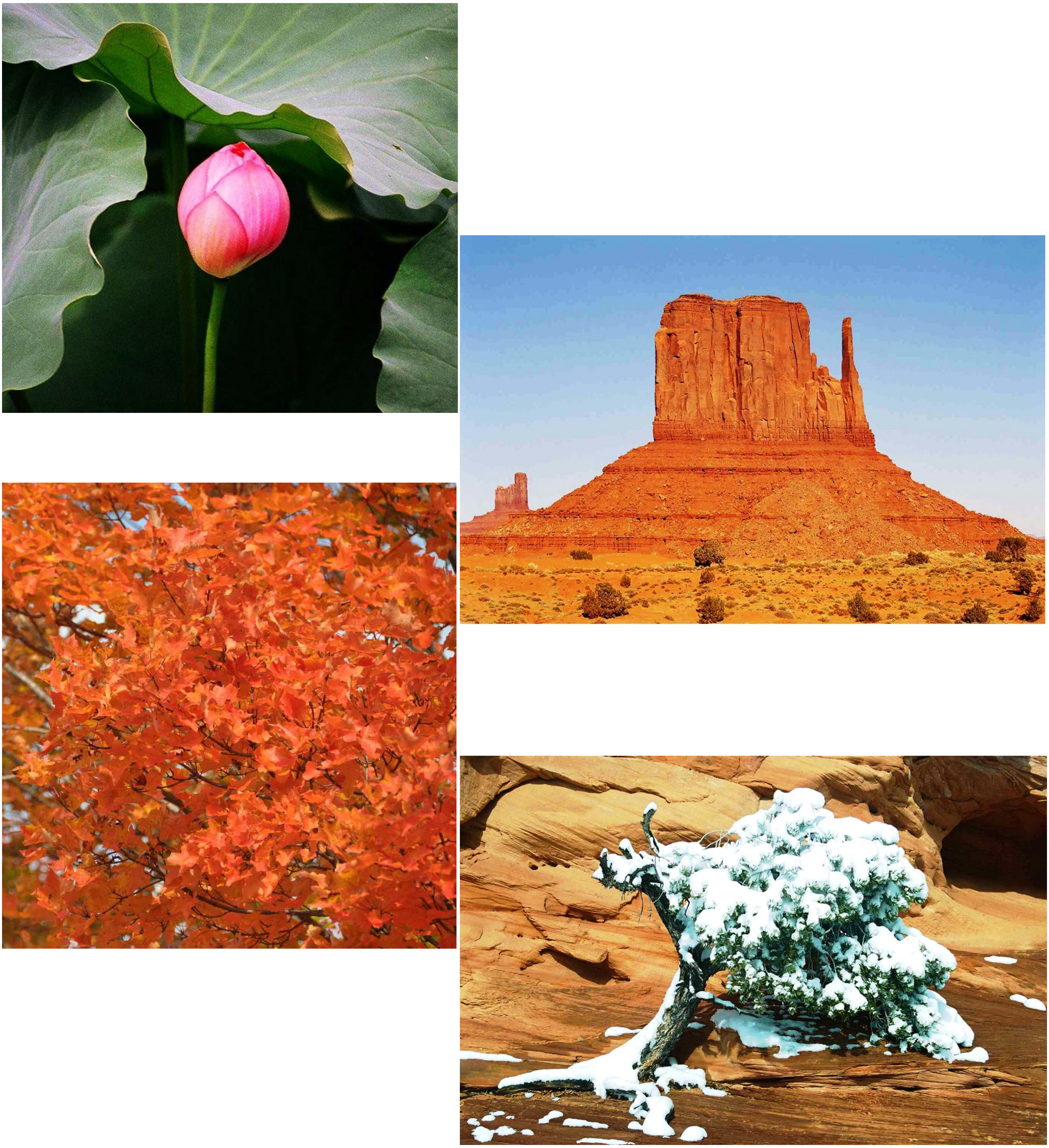


Photo 5.4: Spring – Lotus flower, *Nelumbo nucifera* (Tokyo, Japan); Summer – Monument Valley (Utah, USA); Autumn – sugar maple, *Acer saccharum* (Vermont, USA); Winter – Pine tree (Canyonlands, Utah, USA). (Source: PA.)

- An important property of any location on the surface of Earth is the amount of sunlight available on a given day. We will model this using *daytime*, defined as the time between sunrise and sunset. (This is independent of clouds or weather events.)
- Daytime lengths vary through the year. Some features of daytimes include:
  - The **summer solstice** and **winter solstice**, which are the days with the longest and shortest daytimes (respectively).
  - The **vernal equinox** and **autumnal equinox**, which are the days in spring and autumn (respectively) with daytimes of exactly 12 hours.

Question 5.2.1

Explain why daytimes vary between locations, and from day to day (which is closely related to the reason for seasons). Include solstices and equinoxes in your answer. (Hint: Earth has a tilt of 23.45 degrees on its axis of rotation.)

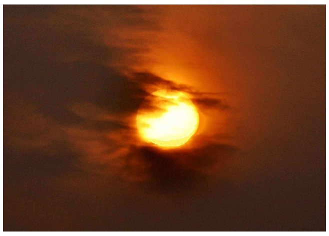
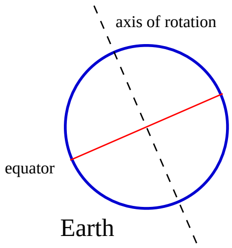
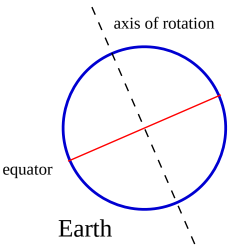
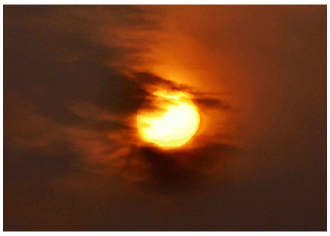


Photo 5.5: Sun and clouds, Shanghai, China. (Source: PA.)







Question 5.2.2

Discuss the daytime lengths in midsummer and midwinter in each of:

(a) Brisbane;

<u>midsummer:</u>

<u>midwinter:</u>

(b) Singapore (which is very close to the equator); and

<u>midsummer:</u>

<u>midwinter:</u>

(c) Santa Claus village, Rovaniemi, Finland (north of the Arctic Circle).

<u>midsummer:</u>

<u>midwinter:</u>

Photo 5.6: Top left: road sign to Santa (Rovaniemi, Finland). Right: the official home of Santa (Santa Claus Village, Finland). Bottom left: Singapore. (Source: PA.)

- At large distances from the equator, summer daytimes are very long; on some occasions there is no sunrise or sunset for a period greater than one day. For simplicity, in such cases we say that the daytime is 24 hours.
- Similarly, in midwinter we say that the daytime is 0 hours.
- Figure 5.4 shows the daytimes in Brisbane at weekly intervals from Friday 1/1/2010 to Friday 31/12/2010.[1] The graph of daytime lengths in every year will be very similar; clearly, the graph resembles a sine wave!

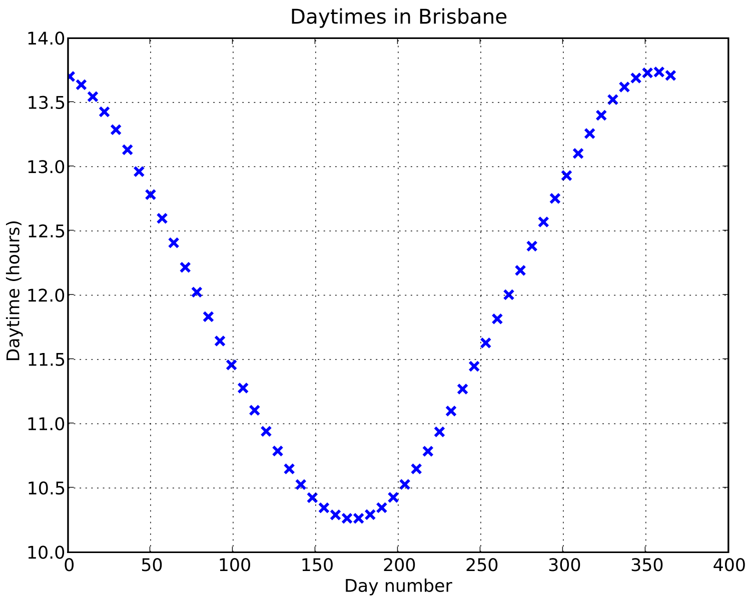


Figure 5.4: Daytimes in Brisbane over the year.

***Question** 5.2.3*

Use the graph in Figure 5.4 to answer the following questions.

(a) When are the solstices in Brisbane, and how long are the daytimes?

(b) When are the equinoxes in Brisbane?

[1]Daytimes were found by subtracting the sunrise time from the sunset time. Sunrise time is defined as the time at which any part of the sun is first visible on a clear, cloudless day. Sunset time is defined as the time at which any part of the sun is first **not** visible on a clear, cloudless day. The definition of sunset differs slightly from standard usage.

Question 5.2.4

On some international flights, in-flight maps show areas of night and day superimposed on the surface of Earth. An example of such a map is shown in Figure 5.7.

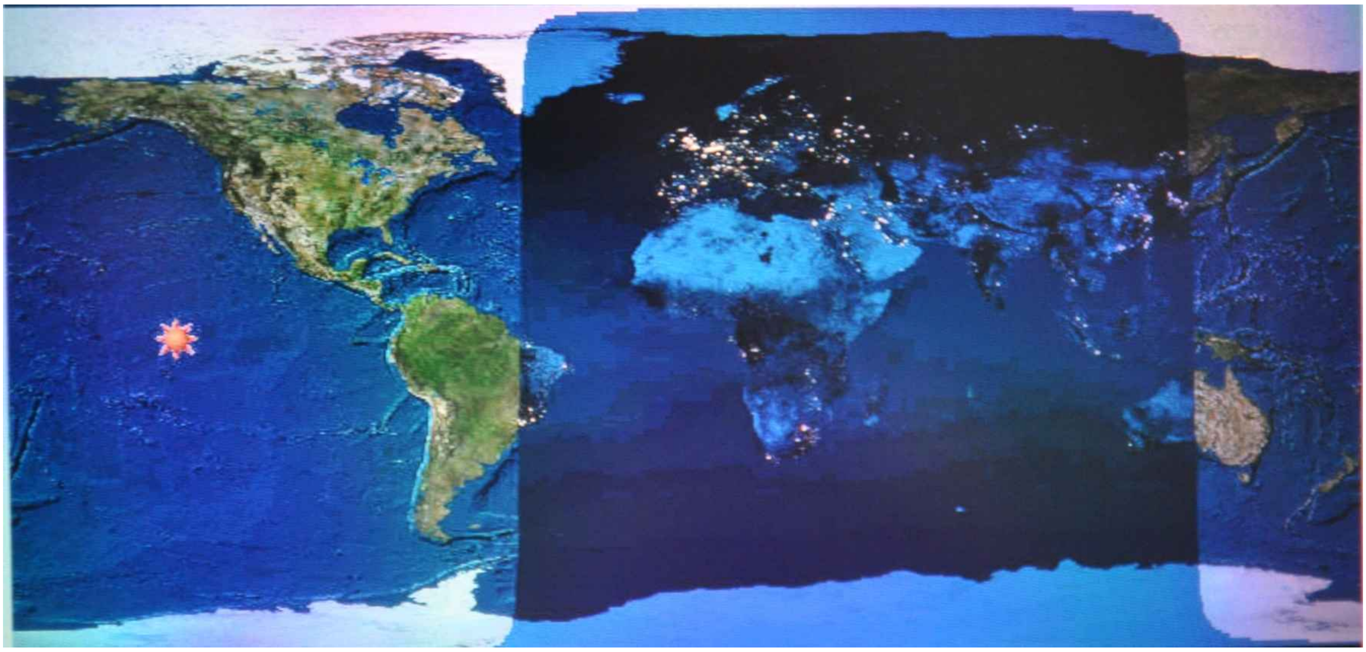


Photo 5.7: In-flight map. (Source: PA.)

(a) On (roughly) what date was Photograph 5.7 taken? Why?

(b) Describe how the map would appear on the winter solstice in the southern hemisphere. Explain your answer.

(c) Describe the map would appear on the March equinox. How would it appear on the September equinox? Justify your answers.

Case Study 13: Modelling daytimes

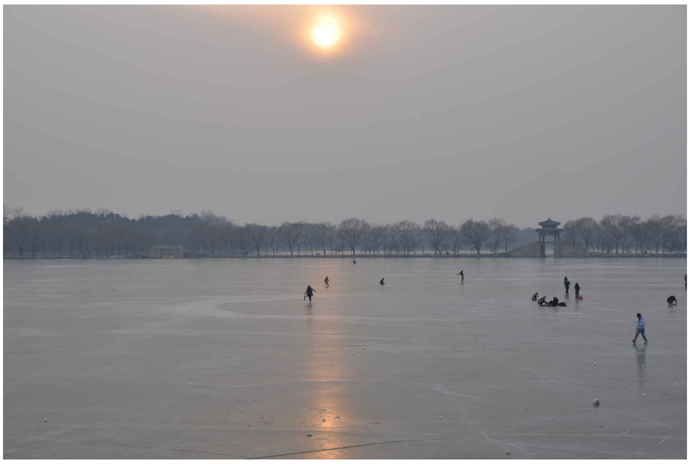


Photo 5.8: Sunrise over Kunming Lake in winter, Beijing, China. (Source: PA.)

Photo 5.9: Sunset over prison guard tower, near Krakow, Poland. (Source: PA.)

- Every location on Earth has a *latitude*, describing its distance from the equator. On any given day, **every location with the same latitude has the same daytime length.**
- At each location on Earth the daytimes form a repeating, yearly pattern, so can be modelled using sin, as a function of the day of the year.
- (In reality, daytimes will vary slightly from these functions as days are discrete time steps whereas the Sun and Earth move continuously.)

Question 5.2.5

If t is the day number in the year (starting from $t = 0$ on January 1st) then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point. At the equator $K \approx 0$, and its value increases for more southerly locations. For Brisbane $K \approx 1.74$; the graph with $K = 1.74$ is plotted in Figure 5.5.

(continued over)

Question 5.2.5 (continued)

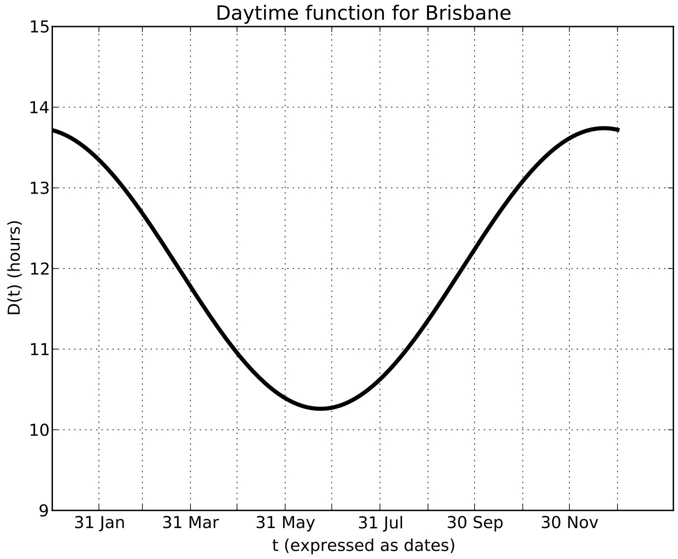


Figure 5.5: The daytimes function vfor Brisbane.

Recall that, in Brisbane, $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t - 264)\right)$.

Discuss the physical and mathematical significance of each term in $D(t)$.

Question *5.2.6*

Recall that, in Brisbane, $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t - 264)\right)$.

Answer each of the following. (The questions are very similar to Question 5.2.3. Now, use the function rather than the graph to answer them.)

(a) When are the solstices in Brisbane, and how long are the daytimes?

(continued over)

Question *5.2.6 (continued)*

Recall that, in Brisbane, $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t-264)\right)$.

(b) When will the solstices occur in Townsville (north of Brisbane) and in Hobart (south of Brisbane)? Why?

(c) The equinoxes have daytimes of length 12 hours everywhere in the world. When are the equinoxes?

Question 5.2.7

In $D(t)$, $K \approx 1$ for Townsville, $K \approx 1.74$ for Brisbane, and $K \approx 3.3$ for Hobart. The graph for Brisbane is shown in Figure 5.6.

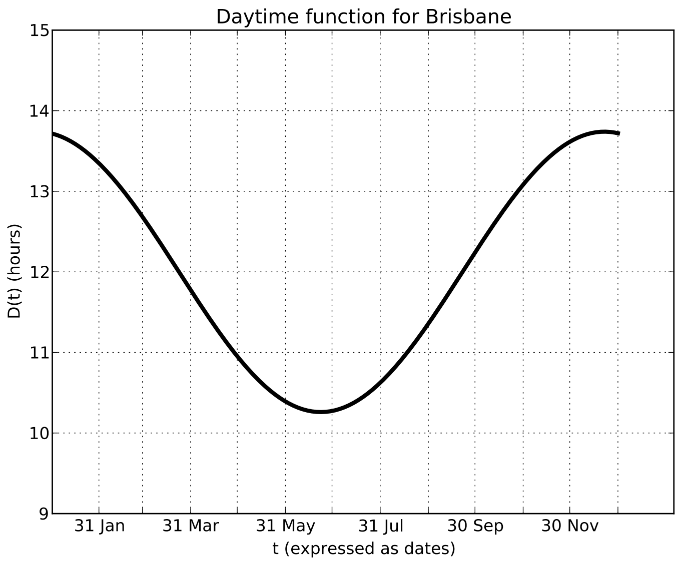


Figure 5.6: Daytimes in Brisbane over the year.

(a) Roughly sketch the graphs of $D(t)$ for Townsville and Hobart on the above graph.

(b) By how much is the daytime on the **summer** solstice in Hobart **longer** than in Townsville? What is the difference on the **winter** solstice?

(c) What does your answer suggest for the total amount of daytime in a year at any location in the southern hemisphere? Is it true, and what does it mean?

Question 5.2.8

Recall that the daytime equation in the southern hemisphere is

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right).$$

The corresponding equation in the **northern** hemisphere is

$$N(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 81)\right).$$

(a) With reference to these equations, explain the similarities and differences between daytimes in the northern and southern hemispheres.

(b) A graph of $D(t)$ is shown in Figure 5.7. Sketch a rough graph of $N(t)$, explaining your answer and identifying the solstices and equinoxes.

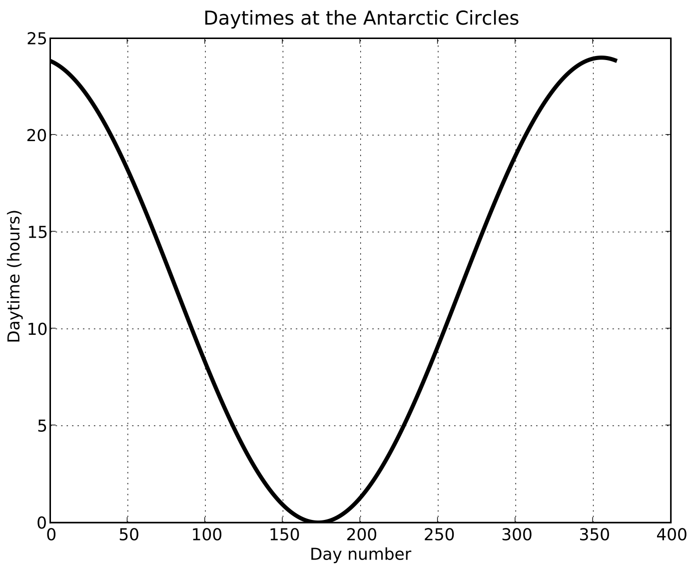


Figure 5.7: Daytimes at the Arctic and Antarctic Circles.

End of Case Study 13.

Case Study 14:
To everything, there is a season, tern, tern, tern.

- Many animals undertake *migration*, during which they move from one area to another, and then return. This often happens on an annual basis, according to seasons or weather patterns.
- Migratory behaviour occurs in all major animal groups (birds, reptiles, mammals, amphibians, fish, insects and crustaceans); see [10].
- Well-known examples of migration include: wildebeest and zebra on the Serengeti plains in Africa; geese "flying south for winter" in the northern hemisphere; salmon returning to their home sream for spawning; humpback whales travelling north along the Queensland coast during winter; and sea turtles returning to beaches to lay eggs.

Question *5.2.9*

What are some of the reasons for, and benefits of, seasonal migration? How does this relate to daytimes?

Photo 5.10: Migrating Canada Geese, *Branta canadensis*, New York State, USA. (Source: PA.)

- The Arctic tern, *Sterna paradisaea*, is a seabird that migrates annually from its breeding grounds in the Arctic to the Antarctic and back.

Image 5.2: Arctic tern in flight. (Source: en.wikipedia.org)

- Individuals have been tracked travelling a distance of 400–700 km per day, and 80000 km in a year; this is the longest (known) migration of any animal.
- Figure 5.8 shows tracked migration routes of 11 Arctic terns (see [12]).

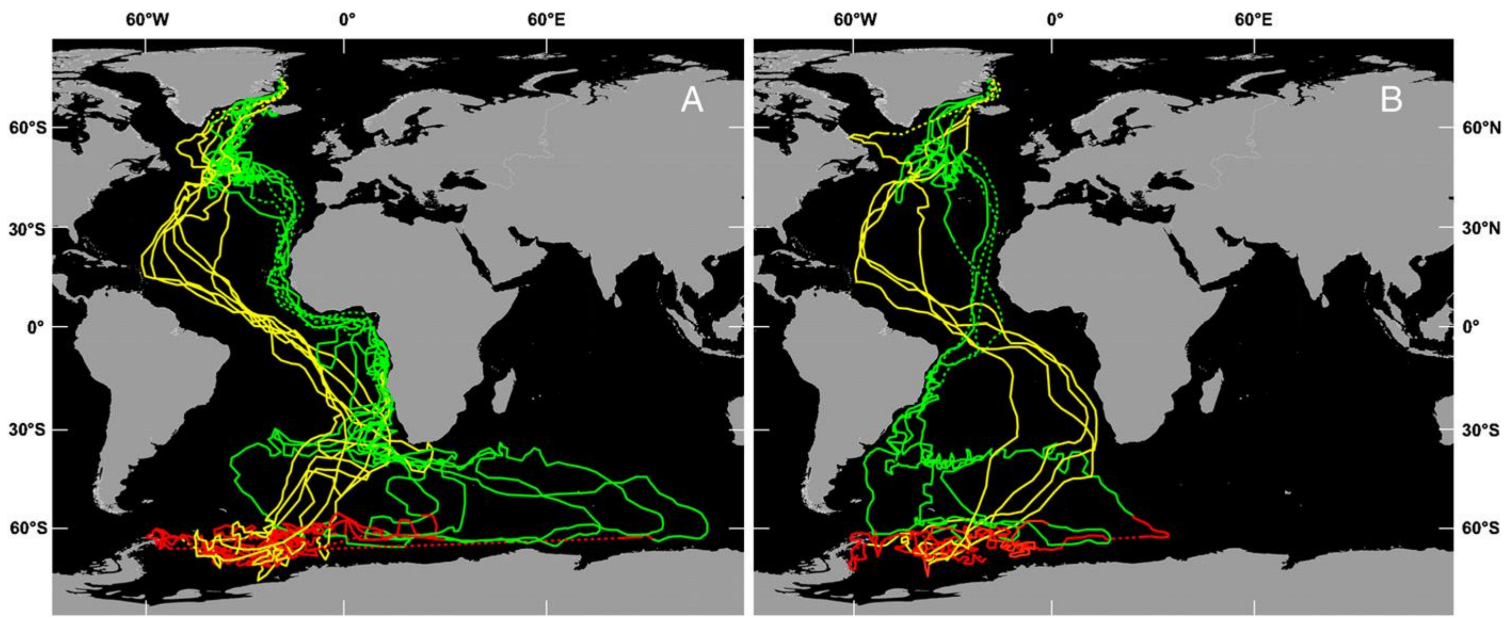


Figure 5.8: Interpolated geolocation tracks of 11 Arctic terns tracked from breeding colonies in Greenland ($n = 10$ birds) and Iceland ($n = 1$ bird). Green = autumn (postbreeding) migration (August/November), red = winter range (December/March), and yellow = spring (return) migration (April/May). Two southbound migration routes were adopted in the South Atlantic, either (A) West African coast (n = 7 birds) or (B) Brazilian coast.... (Reproduced from [12].)

- To collect the data presented in Figure 5.8, researchers in [12] tracked the migration routes of individual birds by attaching miniature archival light loggers to the legs of 20 breeding birds in Iceland in 2007, and to 50 breeding birds in Greenland in July 2007. A year later, data were retrieved from 11 birds.

- The light loggers recorded and stored information about the ambient light intensity at different times and dates, which the researchers used to calculate the time of sunrise and sunset on each day, and hence the daytime.

- They then used the formulae for $N(t)$ and $D(t)$ defined in Question 5.2.8, along with the day numbers and measured daytimes, to calculate the value of K and hence the latitudes of the locations at which readings occurred.

- Next, the calculated latitude and times of sunrise/sunset were used to find the longitude of each reading, and hence pinpoint the location of the bird on each day. Finally, researchers assumed that the birds flew in direct lines between each consecutive pair of readings.

- The paper [12] states that:

  > "Locations were unavailable at periods of the year when birds were at very high latitudes and experiencing 24 h daylight. In addition, only longitudes were available around equinoxes, when day length is similar throughout the world. Overall, after omitting periods with light level interference and periods around equinoxes, the filtered data sets contained between 166 and 242 days of locations for each individual."

Question *5.2.10*

Arctic terns migrate annually between the Arctic and the Antarctic. Figure 5.9 compares daytimes at the Arctic and Antarctic Circles; the graph shows plots of $D(t)$ and $N(t)$, defined in Question 5.2.8, with $K = 12$.

(continued over)

***Question* 5.2.10** *(continued)*

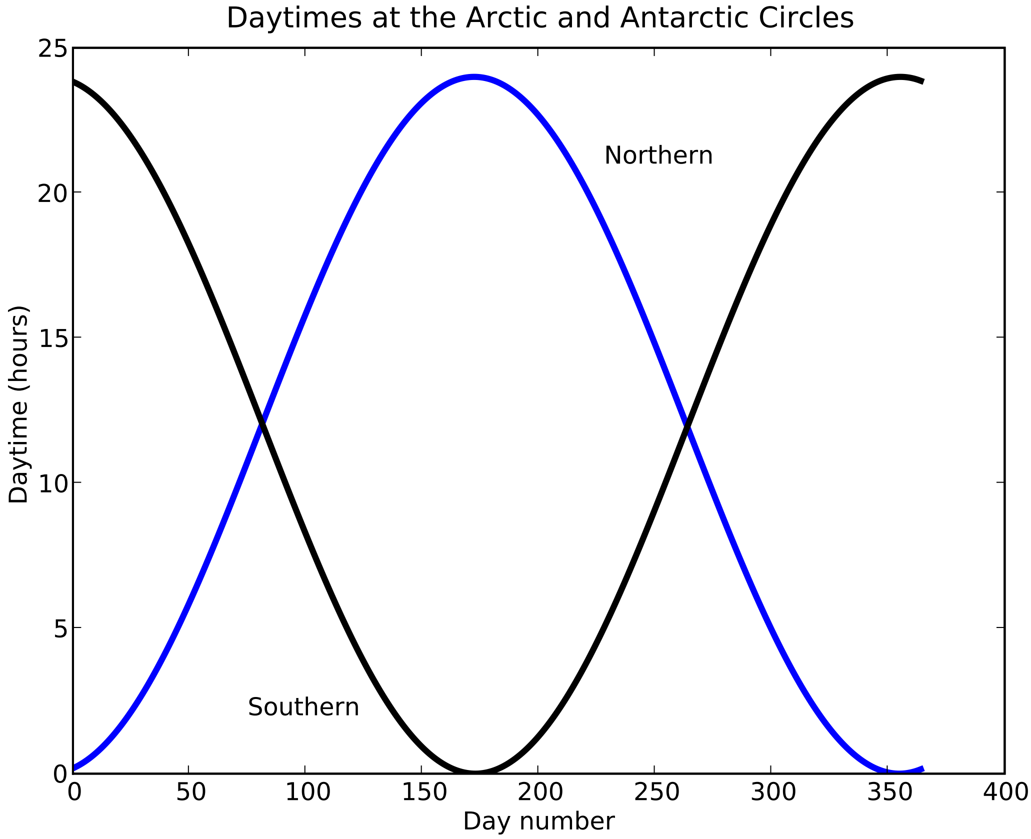


Figure 5.9: Daytimes at the Arctic and Antarctic Circles.

Assume that an individual Arctic tern arrives at its breeding grounds on June 1st (day number 150 of the year).

(a) Roughly how much more daytime is there at the Arctic circle on June 1st than there is at the Antarctic Circle?

(b) On the graph, identify the **total** amount of daytime at the Arctic Circle between June 1 and August 31, and the total **additional** time at the Arctic Circle compared to the Antarctic Circle.

(c) Explain how to calculate mathematically the value in Part (b).

End of Case Study 14.

Question 5.2.11

Earlier, Figure 4.15 showed graphs of the measured summit temperatures and wind speeds on Mount Everest over the period 2002 – 2004.

(a) Figure 5.10 shows the temperature graph, along with the function

$$T(m) = -27 + 9\sin\left(\frac{2\pi}{12}(m-3)\right)$$

where m is the month number from 0 (January) to 12 (next January).

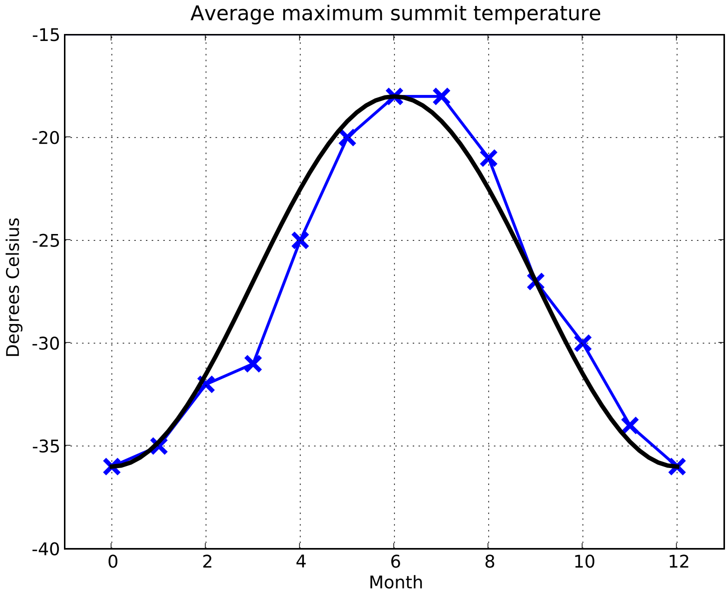


Figure 5.10: Temperatures on the summit of Mount Everest and the graph of $T(m)$.

Explain the physical and mathematical meaning of each term in $T(m)$. How effectively does $T(m)$ model the temperatures?

(continued over)

Question 5.2.11 *(continued)*

(b) Figure 5.11 shows wind speeds on the summit, along with the function

$$W(m) = a + b\sin\left(\frac{2\pi}{12}(m - c)\right)$$

.

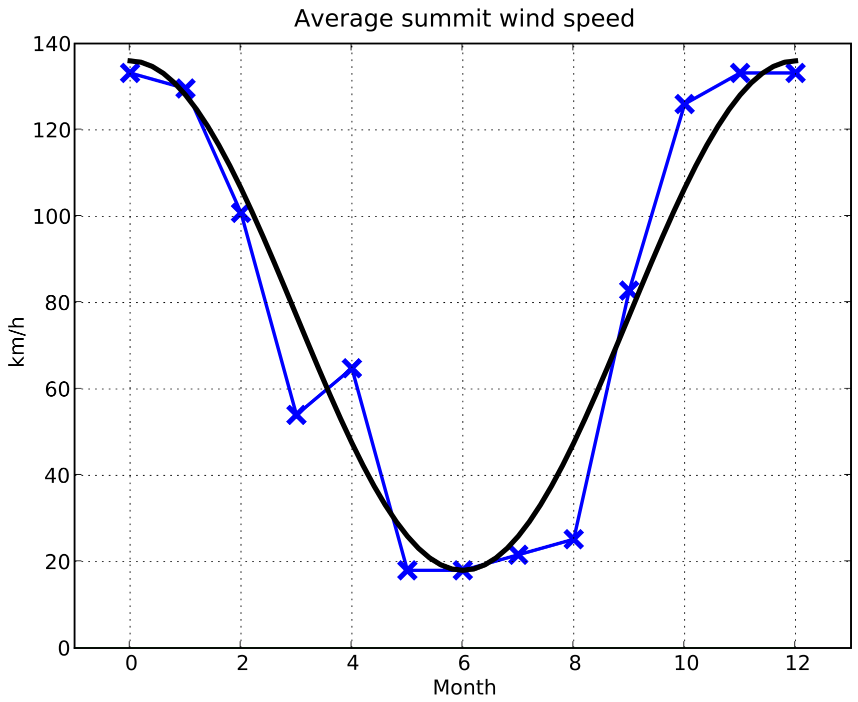


Figure 5.11: Wind speeds on the summit of Mount Everest.

Estimate the values of a, b and c in the equation for $W(m)$.

Question 5.2.12

Fourth Keeling model. Figure 5.12 shows graphs of the Keeling curve and the following function, for t between 0 and 52:

$$y(t) = \frac{1}{3}t^{1.367} + 315 + 3.5\sin\left(2\pi(t + 0.1)\right).$$

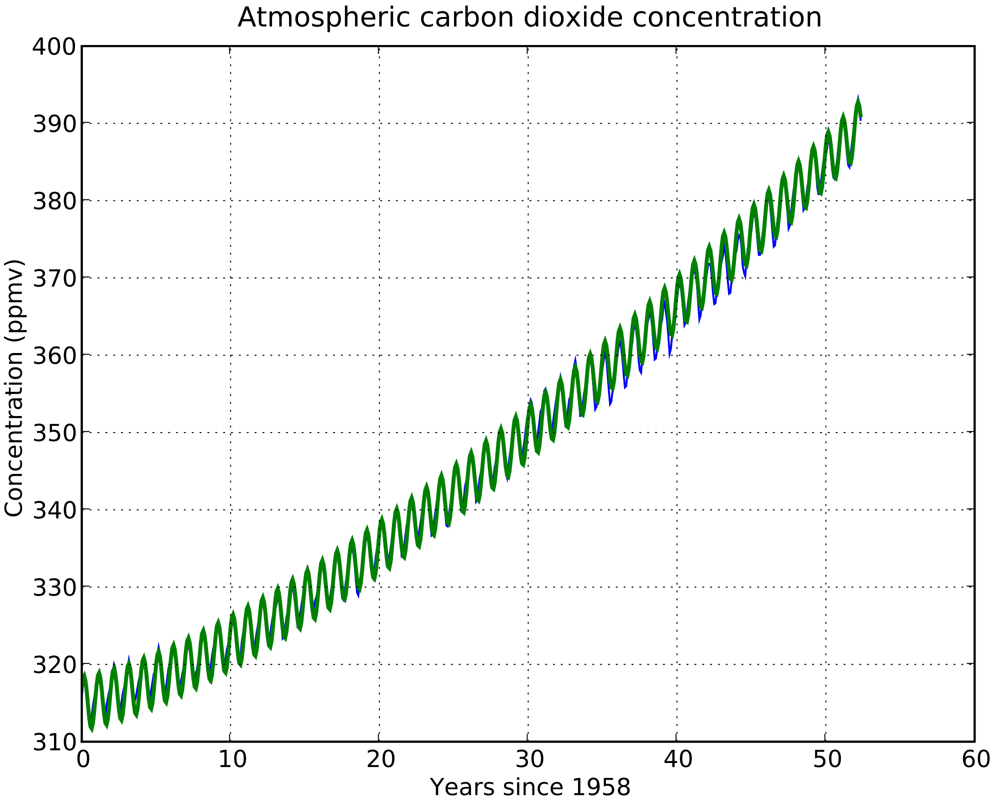


Figure 5.12: The Keeling curve and a model using sin and power functions.

(a) Explain how each term in $y(t)$ impacts on its graph.

(b) How effectively does $y(t)$ model the Keeling curve?

5.3 Space for additional notes

Chapter 6: **Exponentials and logarithms**

Dum dum, diddle dum dum,
diddle dum dum, diddle dum dum.
There was a turtle by the name of Bert
And Bert the Turtle was very alert
When danger threatened him he never got hurt
He knew just what to do. (bang)
He'd duck (quack) and cover, duck (quack) and cover.
He did what we all must learn to do
You and you and you and you. (bang)
Duck (quack) and cover!

Artist: US Federal Government Civil Defense. (www.youtube.com/watch?v=C0K_LZDXp0I)

Image 6.1: *Descent from the cross*, (1435 – 38), Rogier van der Weyden (1399 – 1464), Museo del Prado, Madrid. (Source: upload.wikimedia.org)

6.1 Growth and decay

- Figure 6.1 shows graphs of real, measured data.

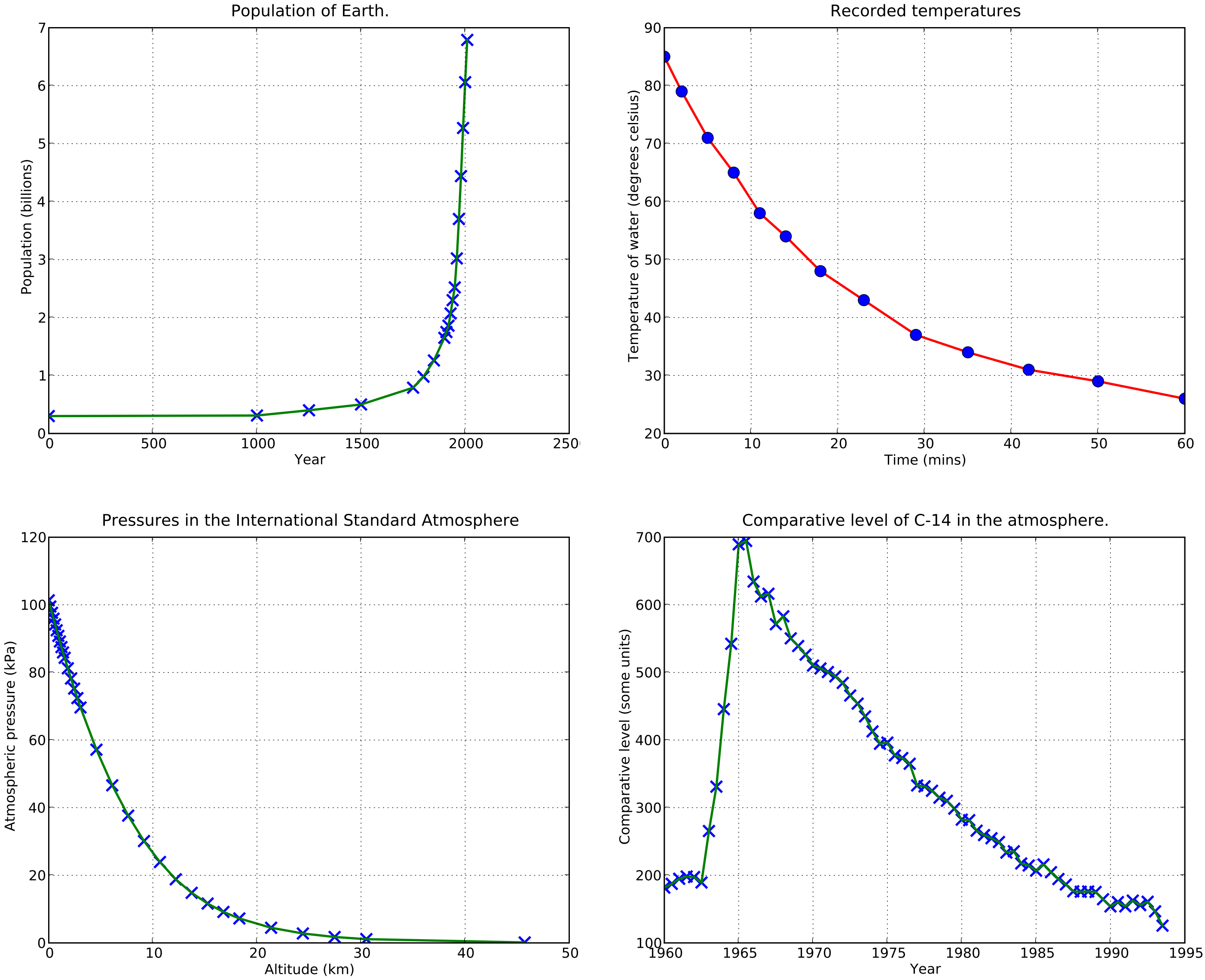


Figure 6.1: Top left: population of Earth over 1000 years. Top right: measured water temperatures in a simple experiment. Bottom left: atmospheric pressures in the international standard atmosphere. Bottom right: comparative level of atmospheric radioactive Carbon-14.

Question *6.1.1*

Why did the level of radioactive Carbon-14 in the atmosphere increase rapidly between 1960 and 1965, and why has it decreased since then?

- In nature, the size, number or amount of most phenomena change over time. Often, the rate of change at any time is proportional to the amount that is currently there.

- Proportional rates of change are typical of many populations. For example, each year the size of the global human population is increasing by around 1.5% of its current size.

- Any phenomenon that has a rate of change proportional to the current amount follows an *exponential* function. (We will see why later.)

Exponential functions

Exponential functions have equations

$$f(x) = Ca^{kx},$$

where C, a and k are constants. The constant a is called the **base**. The two most common values used for the base a are

- the number 10; and
- *Euler's number*, denoted e, where $e \approx 2.71828\ldots$.

Note that:

- when $x = 0$ the function value equals C; and
- the constant k is the **growth rate** or **decay rate**.

- Phenomena that change exponentially can be classified as follows:
  - If they *increase* as x gets larger, they display exponential *growth*.
  - If they *decrease* as x gets larger, they display exponential *decay*.

- Knowing how long it takes an exponential to double in value (for growth) or halve (decay) allows us to study the phenomenon over time.

Doubling time/Half-life

The **doubling time** for an exponentially growing quantity is the time it takes to increase to twice its original size.

The **halving time** or **half-life** for an exponentially decreasing quantity is the time it takes to decrease to half its original size.

Many exponential phenomena in science have relatively constant doubling times or half lives over extended periods.

Growth or decay

Let $f(x) = Ce^{kx}$ where $C > 0$. Then:

- If k is *positive* then the function displays exponential growth.
- If k is *negative* then the function displays exponential decay.

Example 6.1.2

Figure 6.2 shows examples of exponential growth and decay.

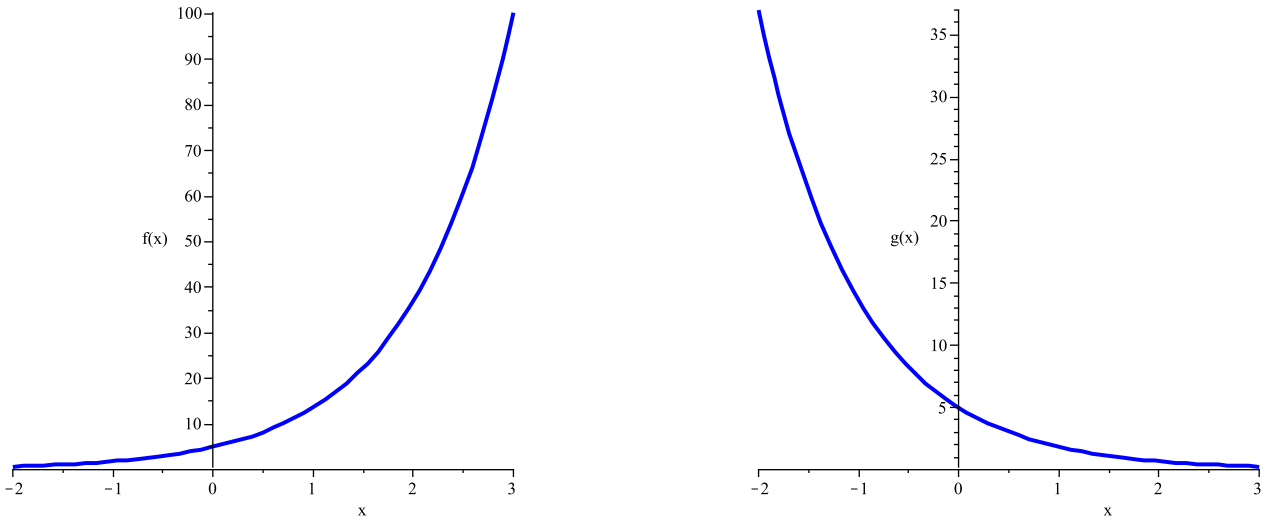


Figure 6.2: Left: graph of exponential growth. Right: exponential decay.

Example 6.1.3

Exponential functions occur frequently in models of nature and the social sciences. Some examples include unconstrained and constrained population growth, radioactive decay and carbon dating, modelling drug concentrations in blood, and modelling *habituation* to a stimulus (in psychology).

- *Logarithms* (or *logs*) are very closely related to exponential functions.

Logarithmic functions

Logarithmic functions are of the form $f(x) = \log_a x$, verbalised as "f of x equals the **logarithm** of x to the **base** a".

In the special case that the base is Euler's number e, then the logarithmic function is often written as $f(x) = \ln x$, verbalised as "f of x equals the **natural logarithm** of x".

Logarithms and exponentials

The relationship between exponentials and logarithms is:

- If $y = 10^x$ then $x = \log_{10} y$ (and vice-versa).
- If $y = e^x$ then $x = \ln y$ (and vice-versa).

Example 6.1.4

Here are some examples of relationships between exponentials and logs.

- $1000 = 10^3$, so $\log_{10} 1000 = 3$.
- $0.01 = 10^{-2}$, so $\log_{10} 0.01 = -2$.
- If $y = e^{0.02x}$ then $\ln y = 0.02x$.

6.2 Exponentials in action

Case Study 15: Radioactive decay

Photo 6.1: The B-29 Superfortress bomber "Enola Gay", National Air and Space Museum, Virginia, USA. (Source: PA.)

- Not all atoms remain the same over time; some undergo *radioactive decay*, which involves rearrangement of the nucleus of the atom, sometimes changing it into a different element.

- When an element undergoes radioactive decay but remains the same element (maintaining the original number of protons), the new atom is called an *isotope*.

- One standard way of denoting isotopes is to write the name or chemical symbol of the element, hyphenated with its atomic mass. For example, Deuterium (an isotope of Hydrogen and the main ingredient in "Heavy water") is written as Hydrogen-2 or H-2.

- Radioactive isotopes have useful applications in a range of sciences and industries, including chemistry, biology, medicine, physics and engineering. Therefore, it is important to understand how to model their decay.

- Radioactive decay is spontaneous, so there is no way of knowing *when* a *specific* individual atom is going to undergo decay.

- However, it *is* known that in any given time period a certain *proportion* of the total quantity in a sample will have decayed.
- Thus, radioactive material undergoes continuous decay at a rate **proportional** to the **quantity** of material, so the decay is an exponential process.

Decay constant

For a radioactive element, the **decay constant** k reflects the rate of decay of the element, and is a property of the chemical element. The half-life can be calculated from the value of k, and vice-versa.

***Example** 6.2.1*

Decay constants and half-lives vary greatly between radioactive elements. For example:

- Polonium-212 has a half-life of about 3×10^{-7} s.
- Uranium-236 has a half-life of about 4.5×10^{9} years.
- Carbon-14 has a half-life of about 5730 years.

***Example** 6.2.2*

Carbon-14 (C-14, also known as *radiocarbon*) is used to determine the age of organic-based artifacts (up to around 60,000 years).

Cosmic rays striking nitrogen in the upper atmosphere produce C-14. It then reacts chemically with oxygen to form radioactive carbon dioxide which permeates living creatures in a fixed proportion, either directly (by absorption from the atmosphere), or indirectly (via food chains).

When an organism dies, it ceases to accumulate C-14, and the remaining amount undergoes net decay over time. *Carbon dating* is the process of measuring the residual level of C-14 in organic artifacts, and thus deducing their age.

Question *6.2.3*

The half-life of C-14 is 5730 years.

(a) Find the decay constant of C-14.

(continued over)

***Question** 6.2.3 (continued)*

(b) Consider the following extract from the paper [9].

> "The Shroud of Turin, which many people believe was used to wrap Christ's body, bears detailed front and back images of a man who appears to have suffered whipping and crucifixion. It was first displayed at Lirey in France in the 1350s . . . Very small samples from the Shroud of Turin have been dated by accelerator mass spectrometry in laboratories at Arizona, Oxford and Zurich. As Controls, three samples whose ages had been determined independently were also dated."

Researchers discovered that 91.9% of the 'expected' C-14 was present. Hence deduce the (approximate) age of the Shroud, and comment on your answer.

End of Case Study 15.

Case Study 16:
Hot stuff, cold stuff

Photo 6.2: Bush fire. (Source: DM.)

- Moving an object with one temperature to a location with a different (but constant) temperature leads to a gradual change in the temperature of the object to match that of the new location.

Photo 6.3: Ice castle, Harbin Ice Festival, China. (Source: PA.)

Question *6.2.4*

Explain why it is reasonable that an exponential function would model the temperature of an object moved to a location with a different temperature.

Photo 6.4: Glass blowing. (Source: PA.)

Photo 6.5: Temperature experiment. (Source: PA.)

Question *6.2.5*

Peter conducted an experiment in which he recorded the temperature of hot water in a container over one hour; the room temperature was 25 °C. Photo 6.5 shows his experimental apparatus and Photo 6.6 shows the recorded temperatures; these temperatures are plotted in Figure 6.3.

(continued over)

Question 6.2.5 (continued)

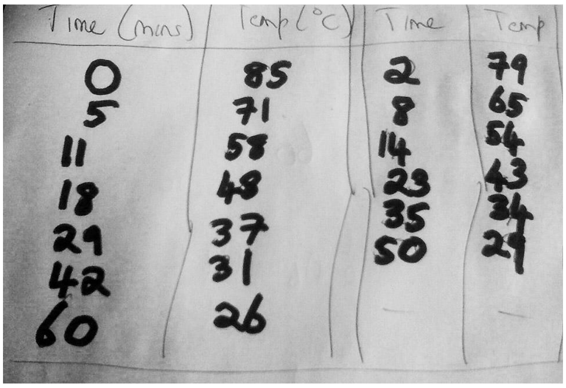


Photo 6.6: Temperature data. (Source: PA.)

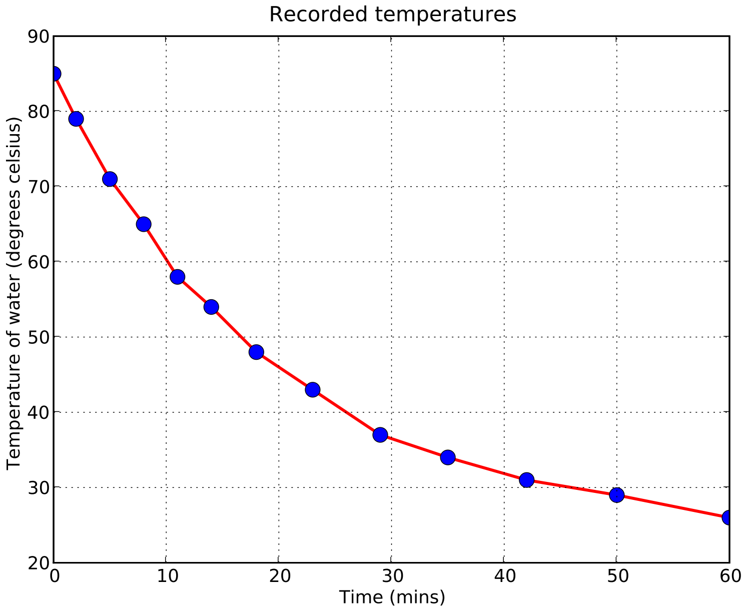


Figure 6.3: A graph of the measured temperatures.

Derive an equation for the water temperature at any time in minutes. (Hint: note that the temperature of the water approaches room temperature of 25 °C, not 0 °C.)

We can develop a computer model to investigate temperature change.

Program specifications: Write a program that plots the measured water temperatures and the function that models the temperatures.

Program 6.1: Temperatures

```python
# Program to plot measured and modelled temperatures.
from __future__ import division
from pylab import *
# Initialise variables
times = array([0,2,5,8,11,14,18,23,29,35,42,50,60])
temps = array([85,79,71,65,58,54,48,43,37,34,31,29,26])
model = 60 * exp(-0.05 * times) + 25
# Draw graphs
plot(times, temps, 'r-', linewidth=2)
plot(times, model, 'k-', linewidth=2)
text(30,40,"model")
text(10,50,"actual")
xlabel("Time (mins)")
ylabel("Temperature of water (degrees celsius)")
title("Recorded temperatures")
grid(True)
show()
```

Output from the program is shown in Figure 6.4.

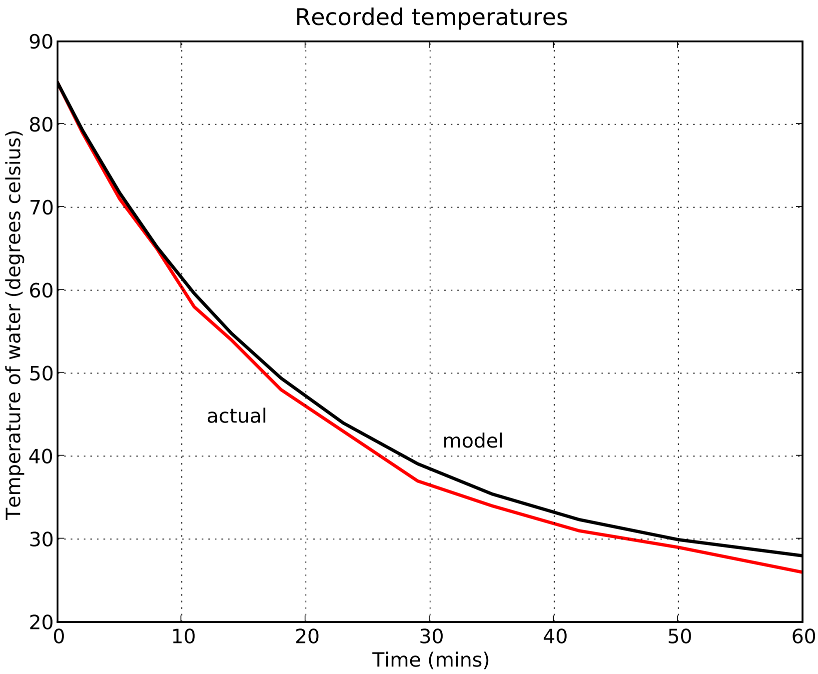


Figure 6.4: Modelled and actual water temperatures.

End of Case Study 16.

6.3 Logarithms in action

- Logarithms provide a convenient mechanism for converting exponential data into a form that can make data analysis easier.

Question 6.3.1

Assume some data are modelled by the exponential function $D(t) = D_0 e^{kt}$. Demonstrate how a logarithmic transformation of the data values results in a linear model. Interpret the y-intercept and gradient of the linear model. (Hint: if x and y are positive then $\ln(xy) = \ln x + \ln y$.)

Question 6.3.2

Earlier we saw that the *International Standard Atmosphere* (ISA) [22] models various atmospheric properties, including temperature, pressure and density. Figure 6.5 shows atmospheric pressures at various altitudes in the ISA, and Figure 6.6 shows a graph of these pressure data transformed using natural logarithm, ln.

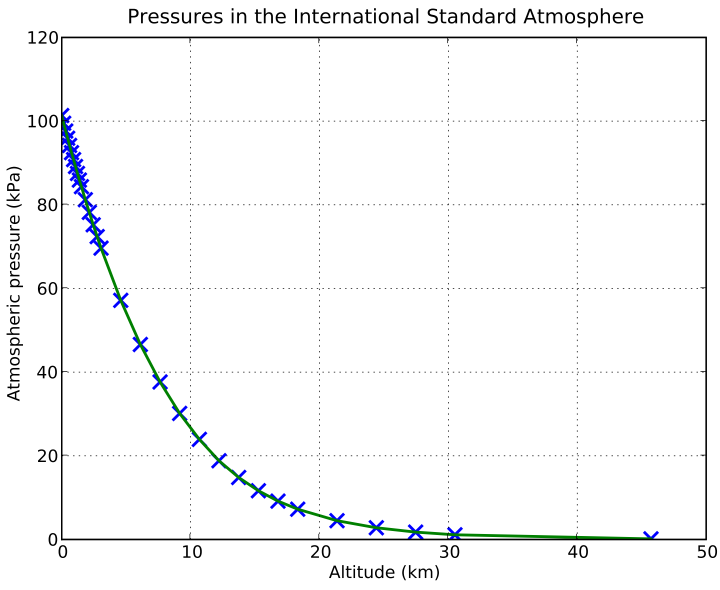


Figure 6.5: ISA pressures.

(continued over)

Question 6.3.2 (continued)

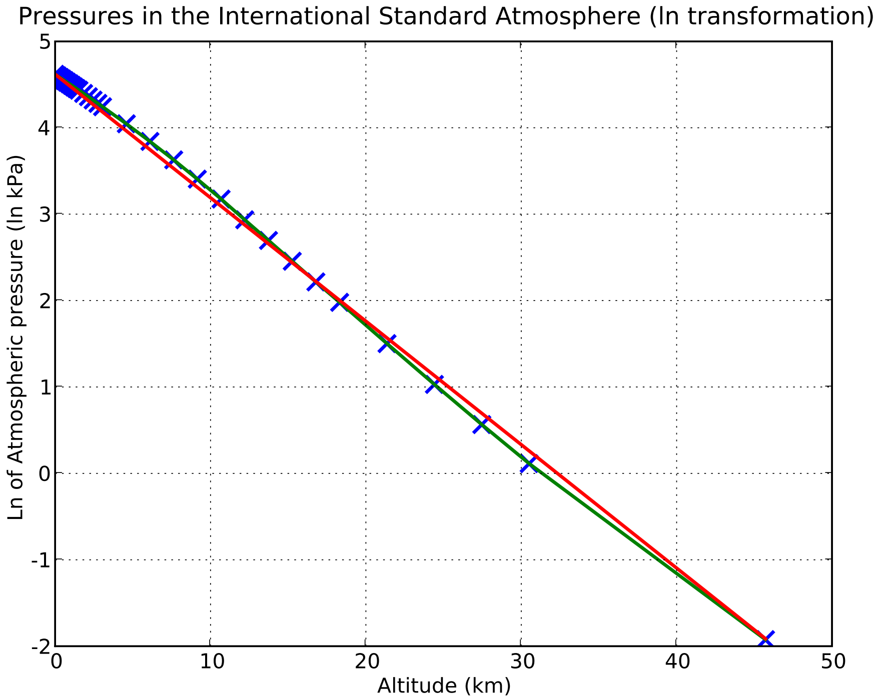


Figure 6.6: ISA pressures (transformed data).

(a) Use Figure 6.6 to find an exponential model of pressures in the ISA.

(b) Hence estimate the pressure outside a jetliner cruising at 10000 m.

Photo 6.7: Bang? (Source: PA.)

Example 6.3.3

In addition to the uses of logarithms we have already studied, some very well-known scientific measurement scales measure log to base 10 of particular quantities. These include:

- *the Decibel scale*, which measures the 'loudness' of sounds (which is directly related to the amplitudes of sine waves);
- the *Richter scale* and *moment magnitude scale*, which measure earthquake intensity; and
- the *pH scale* (discussed below).

Case Study 17: The pH scale

Photo 6.8: Erosion due to acidic rain, Eyam Church, UK. (Source: PA.)

- An important application of logarithms in Chemistry is the pH scale, which is a measure of the *acidity* or *alkalinity* of solutions.
- The pH of a solution reflects its relative concentration of positive hydrogen ions $[H^+]$, in mol/L.
- The pH is defined to equal the *negative of the logarithm to base* 10 *of the concentration*, so

$$pH = -\log_{10}[H^+].$$

- A pH of 7.00 represents a neutral solution, and **decreasing** pH values correspond to an **increase** in acidity. Most substances have pH between 0 (very acidic) and 14 (very alkaline).

Question *6.3.4*

(a) Find the pH of gastric digestive juice in which $[H^+] \approx 10^{-2}$ mol/L.

(b) Find the relative concentration of hydrogen ions in coffee (pH 5) compared with pure water (pH 7).

The rising level of CO_2 in the atmosphere due to greenhouse gas emissions poses a significant risk to the survival of coral reefs. Atmospheric CO_2 dissolves into the ocean and reacts with water to produce carbonic acid (H_2CO_3), leading to ocean acidification, and affecting coral skeletons. Ice core samples suggest that the long-term average pH of seawater was about 8.25. Recent studies predict that the pH of seawater could drop to 7.65 by the year 2100.

(continued over)

***Question* 6.3.4** *(continued)*

(c) If the predictions are correct, what will be the relative concentration of hydrogen ions in sea water in 2100 compared to the long-term average?

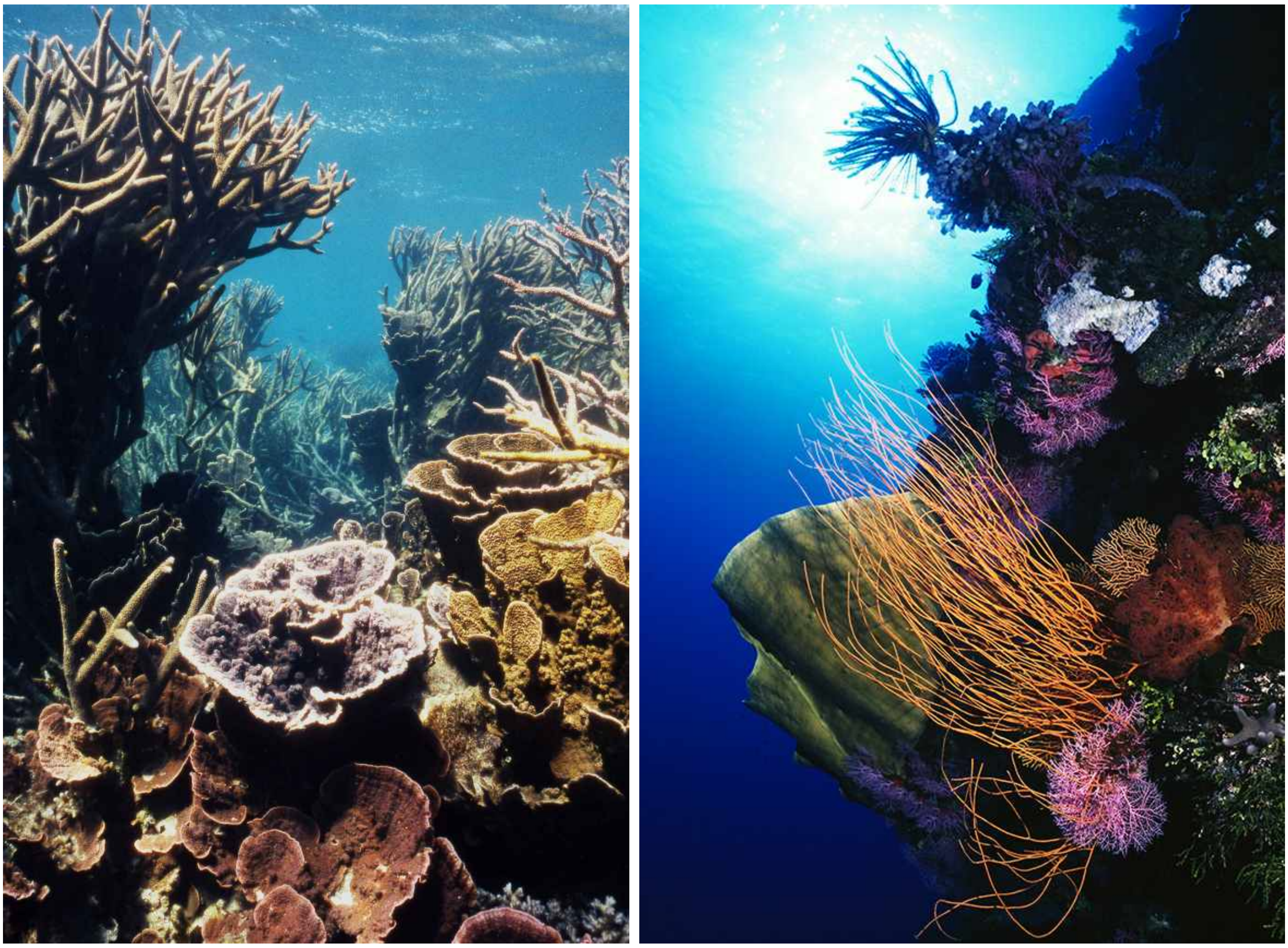


Photo 6.9: Coral reefs. (Source: DM.)

Extension *6.3.5 (from [21])*

"Increases in atmospheric $CO_2 > 500$ ppm will push carbonate-ion concentrations well below 200 μmol kg^{-1} ... and sea temperatures above +2 °C relative to today's values. These changes will reduce coral reef ecosystems to crumbling frameworks with few calcareous corals... Under these conditions, reefs will become rapidly eroding rubble banks such as those seen in some inshore regions of the Great Barrier Reef, where dense populations of corals have vanished over the past 50 to 100 years."

- Image 6.2 (used with permission from O. Hoegh-Guldberg, UQ) illustrates the predicted impact on coral reefs of various levels of atmospheric CO_2 and resultant ocean warming.
- The left image shows the current (comparatively) healthy condition of many reefs. The centre and right images show increasingly degraded reefs, consistent with rising levels of atmospheric CO_2 and resulting ocean warming.

Image 6.2: Predicted impact on coral reefs of rising atmospheric CO_2 levels.

End of Case Study 17.

Question 6.3.6

Fifth Keeling model. Figure 6.7 shows graphs of the Keeling curve and the following function, for t between 0 and 52:

$$y(t) = 280 + 35e^{0.022t} + 3.5\sin\left(2\pi(t+0.1)\right).$$

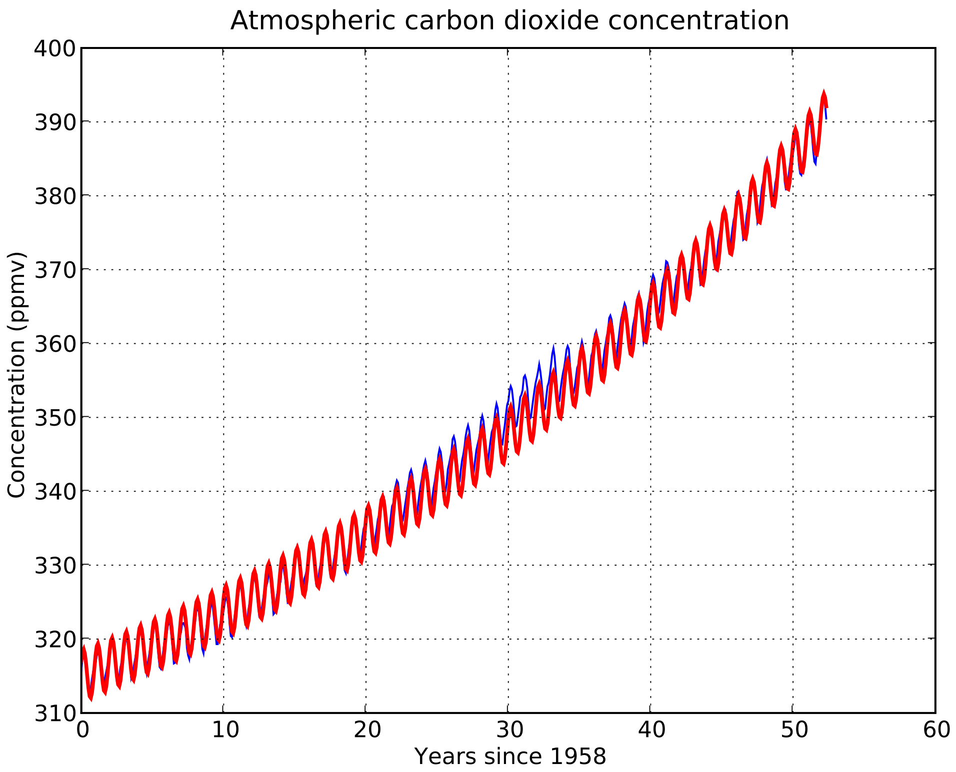


Figure 6.7: The Keeling curve and a model using sin and exponential functions.

(a) Explain how each term in $y(t)$ impacts on its graph.

(continued over)

Question *6.3.6 (continued)*

(b) The equation in Part (a) is

$$y(t) = 280 + 35e^{0.022t} + 3.5\sin\left(2\pi(t+0.1)\right).$$

Earlier we saw that data from ice-core samples show that long-term atmospheric CO_2 levels remained relatively constant at 280 ppm. Use this information and the graph of the Keeling curve to justify the non-periodic component of $y(t)$.

(c) How effectively does $y(t)$ model the Keeling curve?

6.4 Keeling revisited

***Example** 6.4.1*

Over the previous sections we have developed the following three 'good' mathematical models of the Keeling curve.

- Model (1): $y(t) = 0.014t^2 + 0.7t + 315 + 3.5\sin(2\pi(t + 0.1))$.
- Model (2): $y(t) = \dfrac{1}{3}t^{1.367} + 315 + 3.5\sin(2\pi(t + 0.1))$.
- Model (3): $y(t) = 280 + 35e^{0.022t} + 3.5\sin(2\pi(t + 0.1))$.

Figures 6.8, 6.9 and 6.10 each show graphs of the Keeling curve **and an overlay of the corresponding model**.

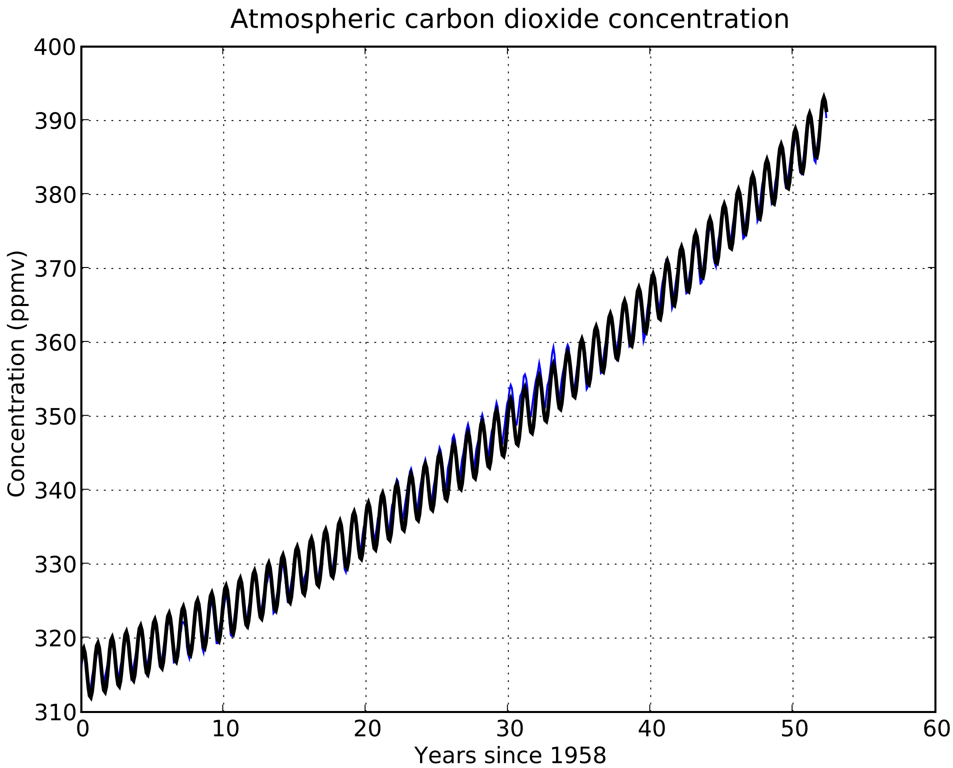


Figure 6.8: The Keeling curve and Model 1 (a quadratic function and a sin function).

(continued over)

Example 6.4.1 (continued)

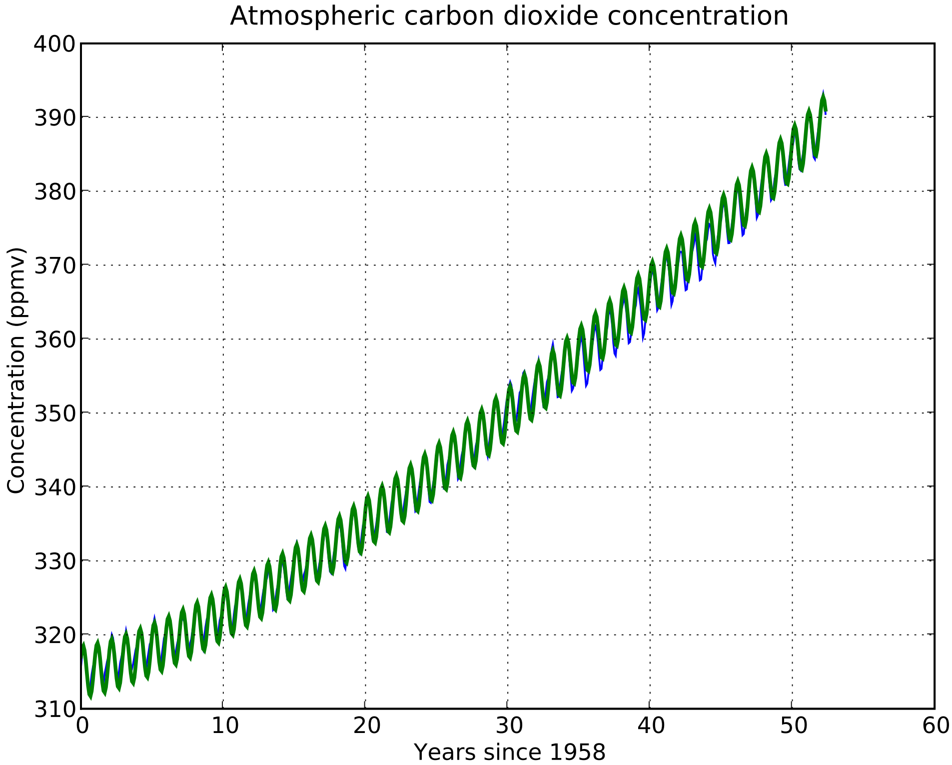


Figure 6.9: The Keeling curve and Model 2 (a power function and a sin function).

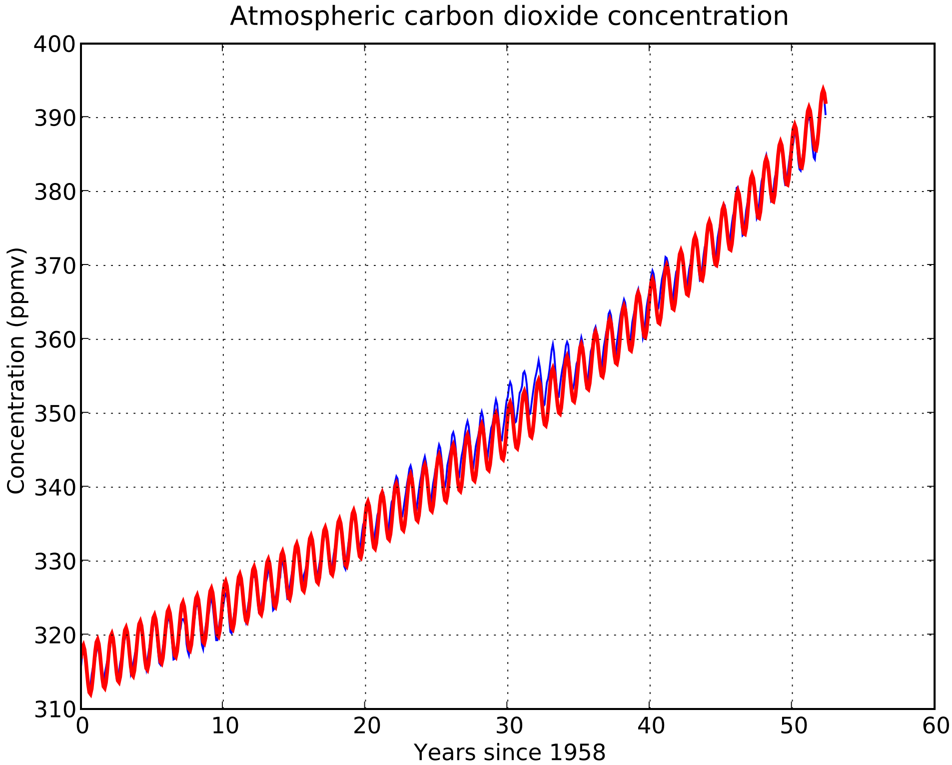


Figure 6.10: The Keeling curve and Model 3 (an exponential function and a sin function).

Question 6.4.2

(a) Which of the three models of the Keeling curve is correct? Why?

The graph in Figure 6.11 shows the three models extrapolated to the year 2058 (100 years after the Keeling study commenced).

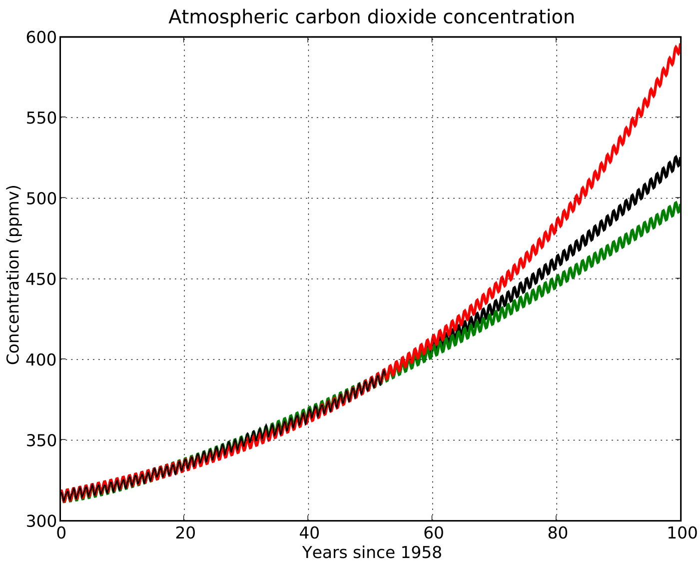


Figure 6.11: The three models of the Keeling curve, extrapolated to the year 2058.

(b) Identify which curve corresponds to each model.

(c) Discuss the ramifications of the different predictions.

(d) What do *you* think the real concentrations will actually look like? Why?

6.5 Space for additional notes

Part 3: Thinking

Tyger Tyger, burning bright,
In the forests of the night:
What immortal hand or eye,
Could frame thy fearful symmetry?

In what distant deeps or skies,
Burnt the fire of thine eyes?
On what wings dare he aspire?
What the hand dare seize the fire?

And what shoulder, and what art,
Could twist the sinews of thy heart?
And when thy heart began to beat,
What dread hand? and what dread feet?

What the hammer? what the chain,
In what furnace was thy brain?
What the anvil? what dread grasp,
Dare its deadly terrors clasp?

When the stars threw down their spears
And water'd heaven with their tears:
Did he smile his work to see?
Did he who made the Lamb make thee?

Tyger Tyger, burning bright,
In the forests of the night:
What immortal hand or eye,
Dare frame thy fearful symmetry?

The Tyger (1794), William Blake (1757 – 1827).

Image 6.3: *Truth, Time and History* (date unknown), Francisco de Goya (1746 – 1828), National Museum, Stockholm, Sweden. (Source: en.wikipedia.org).

Chapter 7: **Quantitative reasoning**

Our galaxy itself contains a hundred billion stars.
It's a hundred thousand light years side to side.
It bulges in the middle, sixteen thousand light years thick,
But out by us, it's just three thousand light years wide.
We're thirty thousand light years from galactic central point.
We go 'round every two hundred million years,
And our galaxy is only one of millions of billions
In this amazing and expanding universe.

The universe itself keeps on expanding and expanding
In all of the directions it can whizz
As fast as it can go, at the speed of light, you know,
Twelve million miles a minute, and that's the fastest speed there is.
So remember, when you're feeling very small and insecure,
How amazingly unlikely is your birth,
And pray that there's intelligent life somewhere up in space,
'Cause there's bugger all down here on Earth.

Artist: Monty Python. (www.youtube.com/watch?v=buqtdpuZxvk)

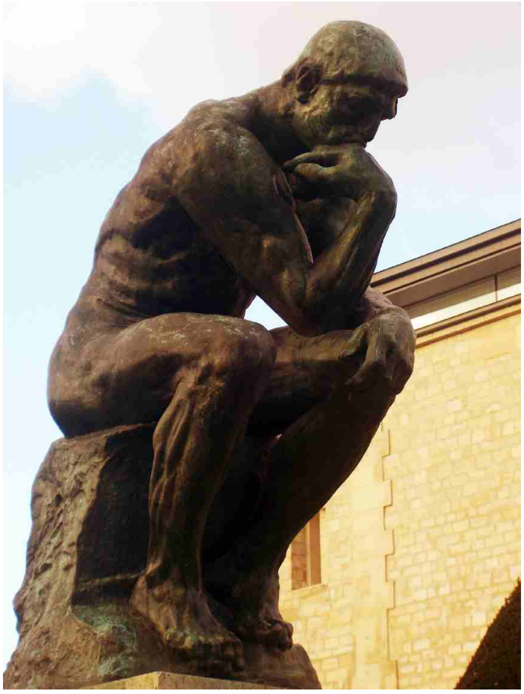


Image 7.1: *The Thinker* (1879 – 1888), Auguste Rodin (1840 – 1917), Musee Rodin, Paris. (Source: en.wikipedia.org)

7.1 Quantitative communication

- In SCIE1000, we will investigate fundamental skills and concepts that will help you to participate in effective scientific analysis and communication.
- We are all producers and consumers of quantitative scientific information:
  - we produce it (for example) in scientific papers, assignments, lecture notes, exam answers and professional communications such as doctor/-patient discussions.
  - we consume it (for example) in scientific papers, the classroom, media reports and when we visit a doctor.
- As a *producer* of such information, we should aspire to be concise, precise, accurate, honest, logical, unambiguous, not excessively technical, and always mindful of the intended audience.
- As a *consumer*, we should aspire to be thoughtful, reflective, sceptical, logical and analytical, while at the same time open-minded and accepting of evidence that may differ from our preconceptions or opinions.
- The media and internet provide a continual bombardment of facts, reports, summaries, interpretations and opinions, often covering sophisticated concepts but written and read by non-experts. In many cases there are errors (or deliberate falsities) in such communications.
- Two approaches useful for identifying errors or false claims are estimation and critical evaluation.
- These approaches should be applied when doing your own work, and also when using material from other sources. They are also useful practices to adopt in everyday life.
- **Estimation** (or *back-of-the-envelope calculations*, or *rough estimation*) is the process of calculating approximate values.
- Estimating involves building rough, conceptual models, then evaluating them either mentally or with simple calculations.

- Estimating 'gives an idea' whether a particular value is plausible. Often, the aim is for the approximate value to be within an *order of magnitude* of the correct value (that is, within a factor of 10).

***Question** 7.1.1*

Develop approaches that allow you to estimate **roughly** answers to each of the following problems, then estimate the value.

(a) Measurements of processes within the body are crucial health indicators. Estimate the total daily volume of blood pumped by your heart.

(b) Estimate the mass of a large storm cloud.

(continued over)

Question *7.1.1 (continued)*

(c) The change in population size over a given time period equals

$$\text{births} - \text{deaths} + \text{immigration} - \text{emigration}.$$

Estimate the number of births in Australia each year.

Critical evaluation

Broadly speaking, *critical evaluation* involves the application of a systematic, reflective and informed sequence of thoughts or actions to investigate a problem, or analyse information.

Question *7.1.2*

What are some key features of critical evaluation?

7.2 Losing patients with mathematics?

- Sometimes, particularly in a medical context, critically evaluating quantitative information is a matter of life and death.

***Question** 7.2.1*

Many SCIE1000 students aim to become doctors, and everyone visits doctors. The Australian Medical Association (AMA) states on its website [2]:

> *"The AMA believes that in order to support and enhance the collaborative nature of the doctor-patient relationship, patients must be able to make informed choices regarding their health care. An informed choice is dependent on receiving reliable, balanced health information, free from the influence of commercial considerations, that is communicated in a manner easily understood by patients."*

Meeting the stated goal places a range of responsibilities on patients, doctors, researchers, medical companies and the media. Discuss these responsibilities from a quantitative science perspective.

Information in [18] is important and relevant to everyone. Key findings include:

- Many people (doctors, patients, journalists and politicians) do not understand health statistics.
- Lack of understanding is due both to lack of knowledge, and intentional misrepresentation of information.
- Sources of medical information (media, information leaflets and journals) tend to overstate benefits and understate risks.
- Commercial and political manipulation undermines informed consent.

The following paragraph is a quote from [18]:

> "Statistical literacy is a necessary precondition for an educated citizenship in a technological democracy. Understanding risks and asking critical questions can also shape the emotional climate in a society so that hopes and anxieties are no longer as easily manipulated from outside . . . "

Question *7.2.2*

Two commonly reported medical statistics are:

- the *5-year survival rate*, which is the percentage of people who are still alive five years after being diagnosed with a condition; and
- the *annual mortality rate*, which is the number of people dying from a given condition each year, often expressed as a rate per 100,000 people.

(a) The 5-year survival rate for prostate cancer in American men is 98%; for British men it is 71%.

(i) Assume that 1,000 British men and 1,000 American men receive a diagnosis of prostate cancer (at the same time). After 5 years how many men in each country are expected to have died?

(continued over)

***Question** 7.2.2* *(continued)*

(ii) Considering only the given statistics, which country has the 'better' health system, and why?

(b) The annual mortality rate for prostate cancer in American men is 26 deaths per 100,000; for British men it is 27 per 100,000. Considering only these statistics, which country has the 'better' health system, and why?

(c) The medical statistics in Parts (a) and (b) are both true. Explain how the (apparent) discrepancies could occur.

(d) Treatment for prostate cancer is invasive with many substantial side effects, including incontinence and impotence. Considering only prostate cancer, which country has the 'better' health system, and why?

Photo 7.1: Freedom square, Brno, Czech Republic. (Source: PA.)

Question 7.2.3

In 1995, an emergency announcement in the UK warned that third-generation oral contraceptive pills doubled the risk of potentially life-threatening blood clots (thrombosis). The announcement lead to widespread concern and fear, and many women ceased using the contraceptives. Reports estimate that in the following year there were an additional 13,000 abortions and 13,000 births, with 800 additional pregnancies in girls under 16 years of age. The announcement omitted the following relevant information:

- the absolute risk of spontaneous thrombosis in young women is around 1 in 10,000.
- the absolute risk of thrombosis when taking second-generation oral contraceptive pills is about 1 in 7000.
- the relative risk of thrombosis increases by a factor of 4 to 8 during a caesarean birth.
- the relative risk of thrombosis during and after pregnancy increases by a factor of around 4.
- the absolute risk of dying from thrombosis during or after an abortion is around 1.1 in 10,000.

Comment on the situation above. What is the significance of these outcomes for doctors and patients? Who benefited from what happened?

Question 7.2.4

In [37], researchers asked 450 American adults (aged 35–70; 320 had attended college; 62 had a postgraduate degree) for answers to the following questions:

> "1. A person taking Drug A has a 1% chance of having an allergic reaction. If 1,000 people take Drug A, how many would you expect to have an allergic reaction?
>
> 2. A person taking Drug B has a 1 in 1,000 chance of an allergic reaction. What percent of people taking Drug B will have an allergic reaction?
>
> 3. Imagine that I flip a coin 1,000 times. What is your best guess about how many times the coin would come up heads in 1,000 flips?"

(a) What are the answers to the above three questions?

(b) What proportion of respondents do you think gave correct answers to each of the questions?

(c) What are the ramifications for doctors, journalists and politicians?

Case Study 18: Breast cancer

- *Breast cancer* develops due to the uncontrolled growth of cells in breast tissue, which enlarge into one or more lumps within the breast. It is a comparatively common cancer, and is a leading cause of death in women.
- Breast cancer predominantly, but not exclusively, affects women.

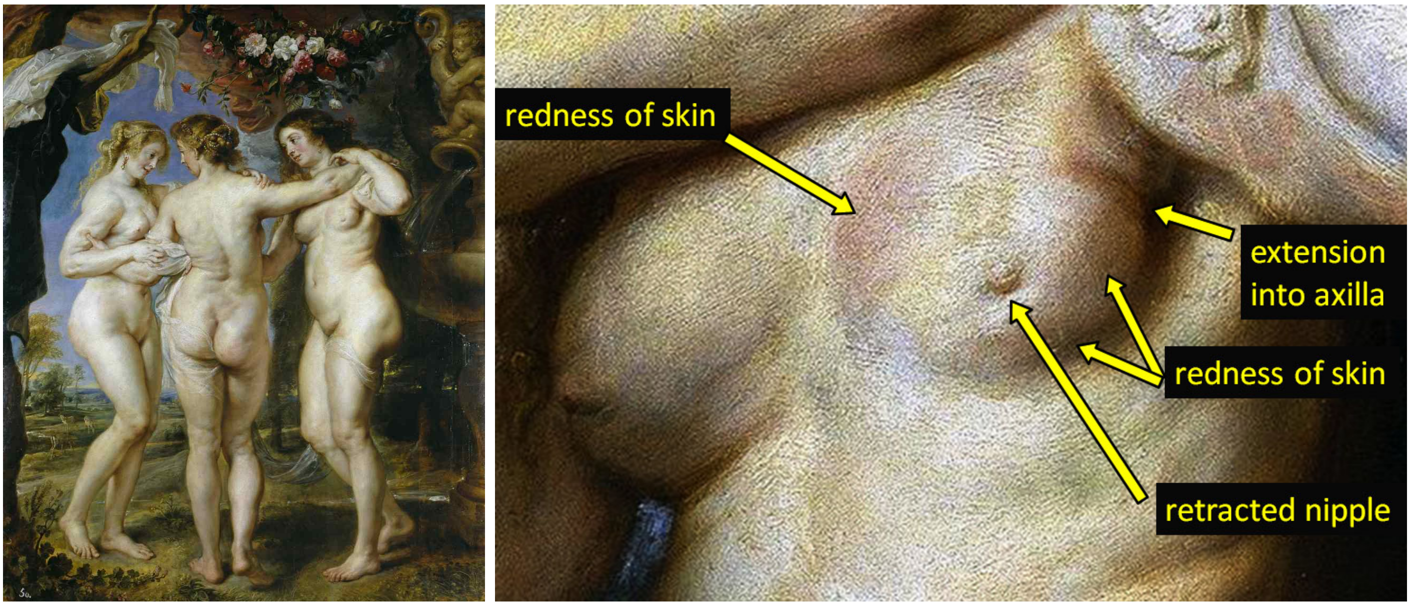


Image 7.2: *The Three Graces* (c1635), Peter Paul Rubens (1577 – 1640), Museo del Prado, Madrid. Top: full painting. Bottom: detail from the left breast of the right-hand Grace. (Source: upload.wikimedia.org)

- A paper [19] states that the right-hand Grace has "a tumor in its external upper quadrant of the left breast which extends up to the left axila [sic]".
- Furthermore, the painting shows a "retraction in the left nipple", that "the total volume of the left breast seems to be smaller than the contra-lateral one", and a "reddness of rounding skin suggesting inflamatory component" [sic].
- The authors conclude that "this is a visual aspect of a locally advanced breast cancer".

A summary of some risk factors for breast cancer given in paper [30] includes:

- gender: the risk for females is around 100 times that for males;
- age: a woman in her 30s has a 1 in 250 chance of developing breast cancer, which increases to 1 in 30 for women in their 70s;
- affluence: breast cancer is more common in affluent societies;
- pre-existing breast conditions (for example, increased breast density);
- hormonal factors (such as age at menopause or oral contraceptive use); and
- high levels of alcohol consumption.

Some factors that reduce the risk of breast cancer include:

- having children (more offspring at an earlier age reduces risk), and breast-feeding
- increased physical activity.

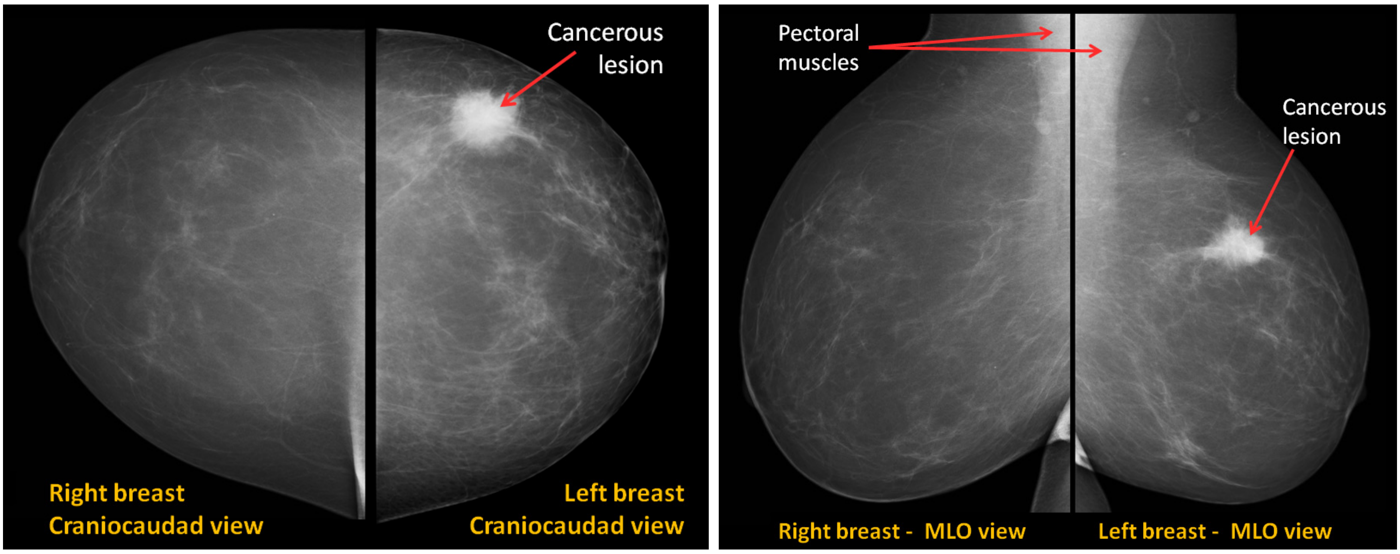


Photo 7.2: Mammographic x-ray images of both breasts in the craniocaudad (head/foot) view (left) and medio-lateral oblique (MLO) view (right). The left breast images show a dense cancerous lesion. (Source: Qld Health and DM.)

Question 7.2.5

A paper [18] quotes an example in which 160 gynaecologists were asked:

"Assume you conduct breast cancer screening using mammography. . . . You know the following information about the women in this region:

(continued over)

Question 7.2.5 *(continued)*

- The probability that a woman has breast cancer is 1% (prevalence)
- If a woman has breast cancer, the probability that she tests positive is 90% (sensitivity)
- If a woman does not have breast cancer, the probability that she nevertheless tests positive is 9% (false-positive rate)

A woman tests positive. She wants to know whether that means that she has breast cancer for sure, or what the chances are. What is the best answer?

A. The probability that she has breast cancer is about 81%.

B. Out of 10 women who test positive, about 9 have breast cancer.

C. Out of 10 women who test positive, about 1 has breast cancer.

D. The probability that she has breast cancer is about 1%."

(a) What is the answer to the above question, and why?

(b) Estimate the proportion of doctors who gave the correct answer?

(continued over)

Question 7.2.5 *(continued)*

(c) What are the implications for you (if you are female), or your mother, sister, girlfriend, daughter or wife?

Image 7.3: Two images of mammographic procedures. (Source: www.cdc.gov, [6].)

- Age is a significant risk factor for breast cancer. Figure 7.1 shows the probability of dying from breast cancer (from [6]).

Age (yrs)	Prob.	Age (yrs)	Prob.	Age (yrs)	Prob.
30	1 in 19180	50	1 in 385	70	1 in 80
35	1 in 4600	55	1 in 230	75	1 in 63
40	1 in 1600	60	1 in 150	80	1 in 50
45	1 in 740	65	1 in 106	85	1 in 43

Figure 7.1: Probability of breast cancer mortality prior to reaching various ages (for females).

Question 7.2.6

The data from Figure 7.1 are graphed in Figure 7.2, along with the function

$$d(t) = \frac{1}{43} \times \frac{1}{55^2} \times (t-30)^2 .$$

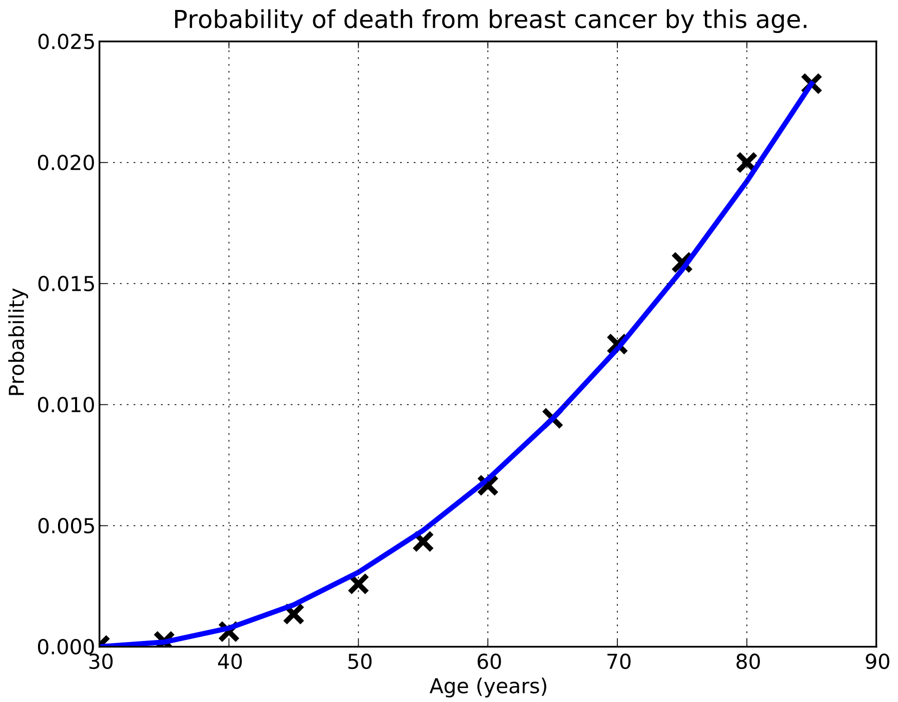


Figure 7.2: Graph of the probability of mortality from breast cancer.

(a) The quadratic $d(t)$ models the probability that a woman will have died of breast cancer prior to reaching age t in years. Explain the physical meaning of each term in the equation for $d(t)$.

(continued over)

Question 7.2.6 *(continued)*

(b) What are the physical implications (for individuals and public health) given that $d(t)$ is a quadratic function?

- Treatment options for breast cancer include: chemotherapy; radiation therapy; hormonal methods (such as anti-oestrogens); and surgery, including total removal of the breast (mastectomy) and breast-conserving surgery (lumpectomy).
- Photo 7.3 shows an x-ray of a breast specimen removed during a lumpectomy. Wires inserted preoperatively during a mammographic biopsy procedure guide the surgeon to the cancerous lesion during the operation. Intra-operative x-rays of the excised breast specimen help to determine whether the removal of the cancerous lesion was successful.

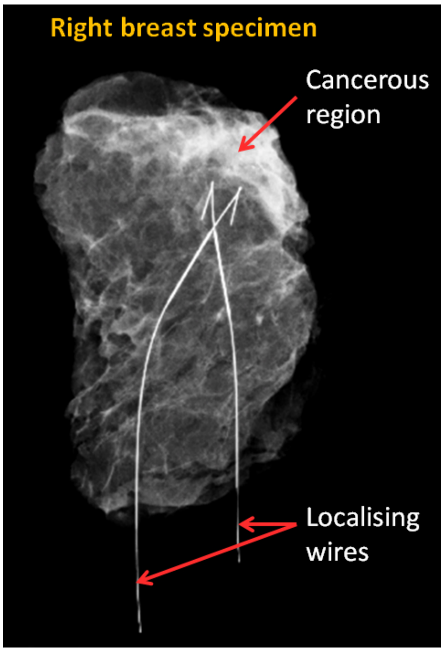


Photo 7.3: X-ray of a breast specimen containing localising hook wires. (Source: Qld Health and DM.)

End of Case Study 18.

Question 7.2.7

In the 1980s, blood screening in Florida found that 22 people who had donated blood tested positive for AIDS. Once notified of the test results, seven of these donors committed suicide. (At that time, AIDs was largely unheard of, and people were not regularly tested. Screening donors for the disease commenced after the discovery that transmission of AIDS occurred through contact with infected blood.)

The AIDS test has a very high *sensitivity* [percentage of infected individuals who correctly test positive] of about 99.9% and *specificity* [percentage of non-infected individuals who correctly test negative] of about 99.99%.

The *prevalence*, or rate of infection, for heterosexual men with low-risk behaviour, is around 1 in 10,000.

(a) What is the (approximate) probability that someone who tests positive for AIDS is infected?

(b) Calculate the probability that at least one person who committed suicide after testing positive did not have AIDS.

Question 7.2.8

(From [18].) To investigate the quality of AIDS counselling for heterosexual men with low-risk behaviour, an undercover client visited 20 public health centres in Germany, undergoing 20 HIV tests.

The client was explicit about belonging to a low risk group, as do the majority of people who take HIV tests. In the mandatory pre-test counselling session, the client asked: 'Could I possibly test positive if I do not have the virus? And if so, how often does this happen?'

The answers from the medical practitioners were:

No, certainly not	False positives never happen
Absolutely impossible	With absolute certainty, no
With absolute certainty, no	With absolute certainty, no
No, absolutely not	Definitely not ... extremely rare
Never	Absolutely not ... 99.7% specificity
Absolutely impossible	Absolutely not ... 99.9% specificity
Absolutely impossible	More than 99% specificity
With absolute certainty, no	More than 99.9% specificity
The test is absolutely certain	99.9% specificity
No, only in France, not here	Don't worry, trust me

Comment on the responses from the German doctors, relating your answer to:

(a) your answers to Question 7.2.7.

(b) the AMA statement in Question 7.2.1 on Page 162.

7.3 Huh?

Question *7.3.1*

Critically evaluate each of the following quoted items.

(a) (Courier mail, November 27, 2009)

"You fall in love, you get married, you have kids – or so the story goes. Sadly, the statistics prove otherwise: one in eight couples in Australia will have difficulty conceiving, and be classified 'infertile'. And while infertility . . . is often perceived as a female problem, it is estimated that in Australia, infertility affects about one in every 20 men. For half of all infertile couples, the problem lies with the male partner, while in 40 per cent of infertile couples using assisted reproduction technologies, the underlying reason is male infertility."

(b) (Australian Vaccination Network publications. Note: the Australian Vaccination Network is opposed to mass vaccinations)

"According to medical reports, children are now less healthy than they have ever been before. More than 40% of all children now suffer from chronic conditions, something that was unheard of prior to mass vaccination. Vaccines have been associated with such conditions as Asthma, Eczema, Food Allergies, Chronic Ear Infections, Insulin Dependent Diabetes, Arthritis, Juvenile Rheumatoid Arthritis, Autism, Attention Deficit Disorder, Ulcerative Colitis, Irritable Bowel Syndrome, Hyperactivity, Schizophrenia, Multiple Sclerosis, Cancer and a raft of other chronic and auto-immune conditions which are experiencing dramatic rises in incidence."

(continued over)

Question *7.3.1 (continued)*

(c) (Courier mail, November 27, 2009)

"HERE is something to get you in the mood tonight: a 10-year Welsh study found that those who enjoyed an active sex life were 50 per cent less likely to have died during that time than those who did not."

(d) (www.naturalnews.com, April 16, 2008) "**Odds of intensive care medication errors are over one hundred percent**

A report produced by PubMed Central states that 1.7 errors per day are experienced by patients in intensive care units (ICU). At least one life-threatening error occurs at some point during virtually every ICU stay. 78% of the serious medical errors are in medications. 1.7 errors per day times 78% equals the likelihood of experiencing a medication error while in an ICU of well over 100% per day. That means the odds are that you will receive the wrong medication or the wrong amount of a medication at least once every single day of an ICU stay."

(e) (www.news.com.au/heraldsun; December 16, 2008.)

"The institute tracked more than 350 patients receiving treatment for back pain. They were followed over one year and contacted at six weeks, three months and 12 months. Dr Maher said the research showed one-in-four would go on to suffer a recurrence of back pain within a year. 'This explains why around 25 per cent of the Australian population suffers from back pain at any one time', he said."

7.4 Space for additional notes

Chapter 8: **Philosophy of science**

Immanuel Kant was a real pissant
Who was very rarely stable.
Heidegger, Heidegger was a boozy beggar
Who could think you under the table.
David Hume could out-consume
Wilhelm Freidrich Hegel,
And Wittgenstein was a beery swine
Who was just as schloshed as Schlegel.

Artist: Monty Python (www.youtube.com/watch?v=m_WRFJwGsbY)

(there is a rude word at time 1:10; song starts at 1:20)

Image 8.1: *The Philosopher in Meditation* (1632), Rembrandt van Rijn (1606 – 1669), Musee du Louvre, Paris. (Source: en.wikipedia.org.)

8.1 What is knowledge, and how is it different from belief?

I believe that Liverpool will win the FA Cup, I believe that I was born in Walgett, and I believe that my four year old daughter is a child genius. Do any of these beliefs count as knowledge? What conditions would have to be met for them to do so? And when we do have knowledge, when can it be said to be *scientific*?

Philosophy of Science involves broad conceptual and critical thinking about the general nature and value of science. Sometimes a look at the history of such thinking can provide a helpful perspective. We will do just that in this module, focusing on the concept of knowledge. We will explore three visions of scientific knowledge, each of which remains relevant today.

8.2 Knowledge – the Platonic Vision

Plato (428–348 BC) was a Greek philosopher who had a vision about the difference between belief and knowledge, and for how knowledge should be our rule of life in society, which he set out in his book *The Republic*. The Greek word for belief is *doxa*. Plato believed that 'right belief', *orthodoxy* in Greek, should rule society (for Plato, a city state) in the sense that we should all hold and share the right beliefs about how the city should be developed and governed. This commitment to orthodoxy is in contrast with other Greek thinkers such as Protagoras (490–420 BC) and Hippocrates (460–370 BC), who believed in *heterodoxy*, the flowering of multiple radical or non-orthodox views. But Plato was aware of the dangers of mere consensus. In Nazi Germany it was orthodox to believe that Jews are inferior, but being a consensus view doesn't make it **right** thinking. It was important to which orthodoxy society subscribed: it must be based on true knowledge. The Greek word for knowledge is *episteme*, from which we get the word epistemology, the study of knowledge.

The Latin translation of episteme is *scientia*, from which we get the word science, although it had a more general meaning, that is, knowledge.

According to Plato, 'true knowledge' is knowledge of unchanging truths, the ultimate reality that lies behind the buzzing, changing world of our experience. Our senses are not the means to gaining such knowledge, rather, it is gained by conceptualising, 'seeing in our mind's eye'. The true nature of a circle, or of justice, the results of geometry, and the ultimate physical principles that explain our world, are only gained by the act of conceptualising in our minds. Like Pythagoras (569–475 BC) before him, Plato thought true reality is mathematical or mathematics-like.

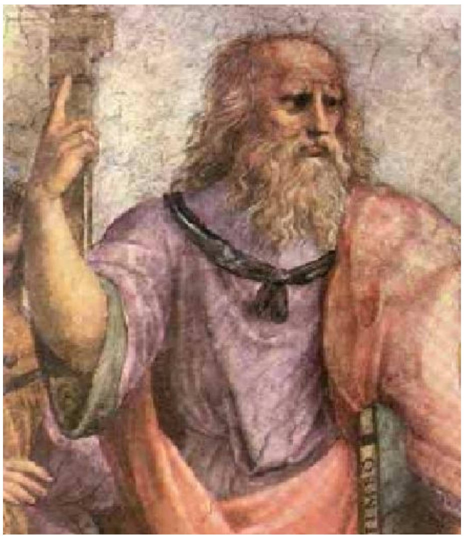


Image 8.2: Plato: "prove it". From *The School of Athens* (1510 – 1511), Raphael (1483 – 1520), Stanze di Raffaello, Apostolic Palace, Vatican. (Source: en.wikipedia.org.)

Our senses, which reveal the buzzing, changing world, do not provide us with true knowledge. They reveal the world of *appearance*. In his 'Allegory of the Cave' (*The Republic* Book VII), Plato describes prisoners chained in a cave, unable to turn their heads, so that all they can see is the wall of the cave. Behind them burns a fire. Between the fire and the prisoners there is a parapet, along which puppeteers can walk. The puppeteers, who are behind the prisoners, hold up puppets that cast shadows on the wall of the cave. The prisoners are unable to see these puppets, the real objects that pass behind them, only shadows and echoes cast by objects Similarly, if all we attend to is the world of our senses, we are like prisoners trapped in a cave. To see beyond appearance we need to conceptualise eternal truths.

To attain such knowledge, those with sufficient aptitude need the right education. Only those who attain this knowledge, episteme, are fit to rule society. Plato called such people *Philosopher Kings*. In ruling, they establish orthodoxy, to which the rest of society should subscribe, since the latter are themselves incapable of much true knowledge.

Two good examples of Plato's vision of knowledge are Euclid (325–270 BC) and Archimedes (287–212 BC). Euclid proved from "self-evident" geometrical axioms and definitions various theorems such as 'the angles of a triangle make two right angles'. Archimedes proved from certain axioms concerning levers, that two unequal weights balance at distances from the fulcrum that are inversely proportional to their weights. Both results involved conceptualising definitions, self-evident axioms, and proofs based on those axioms. Philosophers call this type of reasoning *deductive*, by which they mean an argument whose conclusion cannot be false if its premises (axioms) are true. It was not Plato, but Aristotle (384–322 BC) who set out a system of deductive logic, which remained the best of its kind until the late nineteenth century.

The Platonic vision had a powerful influence among some in the sixteenth and seventeenth centuries, a period of time where many of what we know as the traditional areas of science commenced in earnest, such as Newtonian physics, chemistry, anatomy and astronomy. Rene Descartes (1596–1650) held that true knowledge comes from having "clear and distinct ideas", and utilising those to prove deductively results from self-evident truths. Descartes thought that true knowledge could not possibly be doubted. Evidence of our senses, even of most obvious things like 'this is my hand in front of me' could conceivably be doubted. I don't know for certain that I am not dreaming when I see my hand, or that I am not being tricked by an evil demon into thinking I see my hand. Nevertheless, we can have knowledge of the world around us by deductive reasoning. Mathematical physics deals with quantities to which a number can be attached, and mathematical relations between those quantities can be established beyond doubt, on Descartes' view.

Galileo (1564–1642) also held that mathematical physics enabled us to establish true knowledge that takes us beyond the buzzing confusion of the world of our immediate experience. Galileo clearly understood the significance of idealisation when he wrote [17]:

> Just as the Computer who wants his calculations to deal with sugar, silk and wool must discount the boxes, bales, and other packings, so the

> mathematical scientist when he wants to recognise in the concrete the effects which he has proved in the abstract must deduct the material hindrances, and if he is able to do so, I assure you that things are in no less agreement than arithmetical computations. The errors, then, lie not in the abstractness or concreteness, not in geometry or physics, but in a calculator who does not know how to make a true accounting.

The Platonic Vision was an emphasis, but it didn't mean there was no place at all for experiments. Descartes did a lot of experimental work on human anatomy, and one of Galileo's many contributions was to turn the telescope on the stars to find that there are many more stars than previously thought. But even so, Galileo was a theoretician, and even the experiments for which he is famous were actually thought experiments, such as dropping objects from the leaning tower of Pisa (to show that different objects of different weights fall at the same speed). In theory, the approach of proving theorems from self-evident axioms leaves you with theorems which can be tested in experiment. But if you believe you already have certain knowledge of those theorems, you would not feel any urgency to go and test them.

Key point: the Platonic Vision of knowledge is of mathematical and logical conceptualising, and proofs.

8.3 Knowledge – the Baconian Vision

Francis Bacon (1561–1626) has traditionally been credited as being the 'father of modern science and technology', who 'has permanent importance as the founder of modern inductive method and pioneer in the attempt at logical systematisation of scientific procedure'. He did not share Galileo's and Descartes' appreciation of the importance of mathematics in science, but is famous rather for his vision for experimentation and application.

As suggested by the title of one of Bacon's works *The New Organon*, his account of scientific method and logic was developed with the explicit intention of replacing Aristotle's system of deductive logic. There seems to be a fundamental flaw in a purely deductive system, namely, the so-called problem of *premise regress*. A valid argument tells me that if the premises are true, then the conclusion must be true, but how do I know the premises are true? I could have another deductively valid argument with the first premise as the conclusion. But again, how do I know the premises are true? This leads to a regress. How do we ever reach a starting point - premises which are certainly true, on which knowledge can be built via deductive inferences based on those certain truths? Euclid and Descartes were very clear about what their answer to this problem was. The axioms must be self-evident, beyond any possible doubt. But are there really any such truths? One of Euclid's axioms was that parallel lines never meet, but one can derive a different geometry by dropping this assumption, and in fact Einstein's General Theory of Relativity suggests that our own space-time is non-Euclidean in this way.

Image 8.3: Bacon: "stick to the facts". (Source: en.wikipedia.org.)

Bacon begins the preface to another work (*The Great Insaturation*) with the manifesto:

> "That the state of knowledge is not prosperous nor greatly advancing, and that a different way must be opened for the human understanding entirely different from any hitherto known."

Bacon claimed that the whole scholastic scheme, with its Aristotelian base, was not producing knowledge at all, as evidenced by the fact that it never produced anything of practical benefit for humanity. He thought of the scholastic university as an 'ivory tower', dominated by obscurantist Aristotelian texts and deductive logic, and characterised by a disregard, possibly derived from a Greek disdain for manual labour, for the hands-on knowledge of things of the humble artisan. In the mechanical arts of, say, the silversmith, Bacon saw genuine practical ability and knowledge of the workings of nature.

So, how to attain this new knowledge? Bacon sets out three requirements. The first is a willingness to discard all personal bias, and a desire to know nature as it is, undistorted by theories and presuppositions. Bacon outlines four 'idols of the mind'; habits and ideas which corrupt our capacity for knowledge. The 'idols of the tribe' are tendencies in human nature to accept what we want to believe and what our raw senses tell us, when it suits us, and to our own purposes. 'Idols of the den' are distortions that arise from our particular perspective, 'idols of the market-place' are errors we pick up from each other, often involving the abuse of words, and 'idols of the theatre' are errors associated with grand theories such as Aristotelianism.

The second requirement is to collect all relevant data. In fact, the *New Organon* was a small part of a scheme to produce one huge encyclopaedia of nature incorporating all the available data of observation and experiment. Towards the end of the *New Organon*, Bacon sets out the general plan for what is to be included in this encyclopaedia. For example, suppose we are studying heat and want to know everything about it, free from bias and presupposition. The method involves formulating what Bacon calls the 'Tables of Investigation'. The first Table of Investigation is the 'Table of Affirmation', where everything that contains heat should be listed, according to the 'Rule of Presence': the sun's rays, blood that circulates around the body, certain chemicals, iron after it has been in fire, chilli peppers, and so on. In the second, the 'Table of Negation', everything that does not contain heat should be listed according to the 'Rule of Absence': the moon's rays, the blood in a dead body, or chemicals which are cold. At this point we can formulate a 'Table of Comparisons', in which the different types of data are compared. The 'Prerogative Instances', are twenty-seven ways in which something might stand out when we are studying

a particular case.
For example, the ‘Solitary Instance’ is where two things are similar in many ways, but different in just one way, while the ‘Glaring Instance’ is where there is just one feature of a particular thing that is conspicuous; for example, the weight of quicksilver. In the Preface to the *New Organon*, we find a catalogue of 130 ‘Particular Instances’ by title, including the history of the heavenly bodies, the history of comets, the history of air as a whole, the history of sleep and dreams, the history of smell and smells, the history of wine, the history of cements, the history of working with wood and so on.

Bacon’s third requirement concerns the method for deducing from this collection of facts certain generalisations about nature; that is, scientific laws. For example, in studying heat, we may discover the rule that metals expand when heated. The process will be something like this:

This piece of iron expands when heated

This piece of iron expands when heated

This piece of copper expands when heated

This piece of copper expands when heated

This piece of bronze expands when heated

and so on.

Therefore all iron expands when heated

All copper expands when heated

All bronze expands when heated

and so on.

Therefore all metals expand when heated.

From sufficient observations of iron expanding we draw the conclusion that all iron expands when heated. Then, from the observation that various kinds of metals expand when heated, we conclude that all metals expand when heated.

This method of simple enumeration is one kind of ‘inductive’, as opposed to deductive, inference. Bacon himself did not think simple enumeration was adequate by itself, but that it must be supplemented with reference to the Tables of

Negation and Comparison. So his inductive arguments are more complex than simple enumeration. But for any inductive argument, the premises, particular observations, do not guarantee the truth of the conclusion in the logical sense, since it is logically possible for the premises to be true and the conclusion to be false. The premises simply render the conclusion probable. The problem of premise regress, however, is overcome, since the entire process is grounded in simple particular observations, which, according to empiricism, are the root of all knowledge. So by following the Baconian inductive method, we arrive at generalisations from observation, that is, the laws of nature.

Bacon believed that true knowledge always leads to practical application, since true knowledge of nature gives us power over nature. (Of course, such practical application may not be immediate.) If I understand metal to the point that I know with certainty that heating a certain piece of copper will cause it to expand, then that knowledge gives me power to control it. If I want it expanded, I can heat it. If I do not, I can prevent it from being heated. For example, suppose part of the deck of a ship is made from metal, and I want to prevent expansion because that tends to warp the wood which can cause leaking. I can prevent that expansion by preventing the heating; for example, shielding the metal from the sun if that is the source of heat. In this way Bacon thought that understanding of nature would automatically lead to control of nature, with practical benefit. Knowledge is power. As Bacon claims in the *New Organon*, in a rather self-satisfied tone:

> "I may hand over to men their fortunes, now their understanding is emancipated and come, as it were, of age; whence there cannot but follow an improvement in man's estate and an enlargement of his power over nature."

In *The New Atlantis*, Bacon describes a utopia in which scientists work hard to apply their knowledge to the improvement of the quality of human life. Bacon cites three inventions as evidence that such a utopic vision would be realised if his understanding of science were followed. The first is the printing press, which aids the dissemination of knowledge, the second is gunpowder, an obvious source of power, and the third is the compass, which greatly improves navigation. For Bacon, these three inventions demonstrated conclusively the capacity of scientific knowledge to give power over nature. They lend support to the idea that if we pour our efforts into true science, we will be rewarded with such technological advances, which in turn improve the quality of life. Bacon's optimistic view of human achievement marks the early stages of a trend which dominated Western thought right through until the early twentieth century.

Unlike Descartes and particularly Galileo, Bacon himself did not make much progress with any actual scientific projects. He is seen rather as a philosopher of the scientific method and its technology, who succeeded in specifying the methodology and research program required for successful science. It was not long, however, before the kind of scientific successes that Bacon had hoped for did, in fact, occur. Eighty years after Bacon's death, his philosophy of science was adopted by the Royal Society in London, which set itself up with the explicit aim of carrying out the work that Bacon had envisioned, adopting him as a kind of patron saint. At their meetings, the Royal Society reported on and discussed those experiments, collected data, and so on. Society members included figures such as Boyle, Hooke, and Harvey; in other words, many of the founders of modern science.

Key point: According to the Baconian Vision true knowledge is derived directly from observations and experiments, and will produce practical benefit.

8.4 Knowledge – the Popperian Vision

Karl Popper (1902–1994) was an Austrian philosopher who fled Nazi Germany for New Zealand, and later London. He opposed the Baconian vision on a number of points. First, it doesn't match much of scientific practice. Scientists do not in general conduct experiments without preconceptions. Usually they have a good idea of what they are looking for, and are selective in the facts that they collect. No-one records the name of the cleaner or the colour of the paint on the laboratory wall. Generally theories come first, and the experiments which distinguish them from the alternatives come along later. And second, Popper thought the very mechanism of induction is dubious, as it falls short of a proof. Related to this is the Problem of Induction, first pointed out by David Hume

(1711–1776). This is the problem that, while you can formulate a 'Rule of Induction' which tells you to make generalisations in the right circumstances, you can never prove this rule. It can't be proved mathematically or logically, since it is always logically possible that the next metal you observe, for example, will not expand when heated even though previously all observations suggested that it would. There is no logical contradiction to suppose it doesn't. And secondly, a Law of Induction cannot be proved by experiment, since that proof would itself be an inductive generalisation. That would be to beg the question. You may as well say 'I know my crystal ball is a good predictor because it tells me it is'. So it seems that the use of induction always has an unproved assumption, that nature will continue working the way it always has, as assumption Hume called the uniformity of nature.

Popper therefore proposed an alternative vision of how we come to scientific knowledge. Science proceeds, he said, by *conjectures* and *refutations* [32]. Conjectures are the starting point. They are hypotheses, educated guesses proposed for the purpose of being tested. In fact, the key thing about a conjecture is that it must be *falsifiable*, able to be proved false. According to Popper this is the mark of true science. Any claim that cannot be falsified in principle is not scientific. For example, open today's newspaper and read your horoscope. It probably makes predictions about how your day will go. Now try and think of a set of circumstances that could happen today which, if they did happen, would refute the horoscope's prediction. Often you find this is very difficult, because the claim is not actually falsifiable. So it's not scientific, according to Popper. Popper was a trenchant critic of Marx and Freud, claiming that their theories were meaningless because they were not falsifiable. A theory is not scientific if it can explain everything, no matter how things turn out.

Scientific conjectures should be bold, and clearly able to be refuted. They do not need to be unbiased in any sense. Thinking up bold and novel hypotheses can be a very creative process, and can be prompted by all kinds of things, such as in the case of August Kekule (1829–1896), who said he discovered the ring shape of the benzene molecule after having a day-dream of a snake biting its own tail.

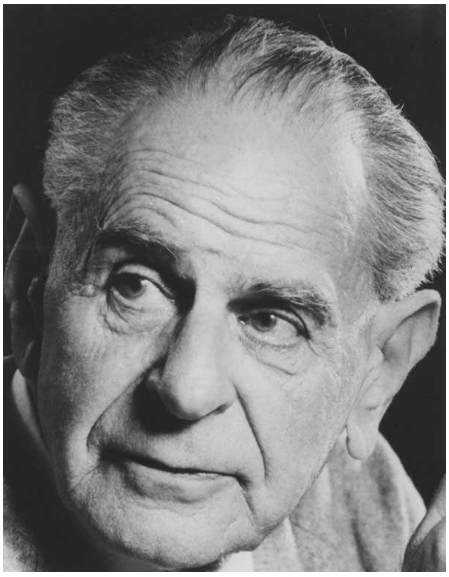


Image 8.4: Popper: "prove me wrong, please". (Source: en.wikipedia.org.)

Once one has a hypothesis, one can deduce the particular results that it predicts, which are then able to be tested. This is not an inductive step, it is deductive. The hypothesis 'all metals expand when heated' entails as a matter of deductive certainty that a particular example of a metal being heated will expand. So there is no problem of induction. Thus Popper's vision is of what is often called the hypothetico-deductive scientific method.

The second key to Popper's vision is that if proved wrong the theory should be immediately rejected. This is scientific progress. At least we know that particular hypothesis is not right. The scientific attitude is to be able to throw out a theory if it is proved wrong. But a theory or hypothesis can be accepted if it survives all attempts to refute it.

Like Plato's and Bacon's visions, Popper's vision has its critics. One problem is that scientists are often looking to test whole theories, or in effect groups of hypotheses 'joined together'. Then, if you refute the group of hypotheses as a whole, the next question would be which part is the part to be rejected. A second problem is that scientists do not always throw out the theory just because there is a problem. If there is no better theory available, it may be held onto, at least for the foreseeable future. Just because a theory has a difficulty with one particular experiment does not mean the theory gets thrown out immediately. And finally, if all we ever have in science is unrejected hypotheses, where is the vision that we ever come to true knowledge in science? On Popper's account we can know a theory is false, but we can never know it is true.

One prominent critic of Popper is Thomas Kuhn (1922–1996), whose book *Structure of Scientific Revolutions* was the most cited book in the twentieth century. According to Kuhn, science goes through different stages historically. There are periods of *normal science*, where scientists are essentially puzzle solving, and periods of *revolution*, where everything is thrown up in the air and completely new theories come to the fore. Normal science is governed by a *paradigm*, which involves certain big theories such as those of Newton, Einstein, or Darwin, together with methodological assumptions, protocols and conceptual elements. Scientists from all around the world work 'together' in that they subscribe to the paradigm.

To take a not-very intellectual example, unlike in Newton's day, today if one writes a scientific paper reporting the outcomes of experiments or experimental studies, one should set out the method so that it can be repeated. Only when the experimental result is reproduced two or three times by independent groups working in different locations is the result accepted as fact. But before it gets to that stage, the paper has to be published in a reputable scientific journal and to achieve that it must be *peer reviewed*, that is, approved by other (usually two) independent scientists. This means normal science is conservative, tending not to accept ideas and approaches that are too radical or unrecognisable from the perspective of the paradigm. Thus to work in normal science you have to be orthodox in Plato's sense. Kuhn did not make these observations in order to denigrate normal science. On the contrary, its conservative nature enables scientists to get on with solving problems and exploring the technological potential of the paradigm. Another feature of normal science is that it permits anomalies, unresolved difficulties. We mentioned above that scientists do not always throw out the theory just because there is a problem. A paradigm can always tolerate a certain amount of anomaly.

However, if the number and the significance of anomalies become too great, the paradigm can enter into a period of crisis, where the tenets of the paradigm can be questioned. This is the beginning of a scientific revolution. Alternative theories and methodologies emerge, and science takes on a more *heterodoxical* look. Eventually, once one of these wins out and a consensus emerges, we enter into another period of normal science with a new paradigm. The new paradigm may be radically different from the old one, to the point that Kuhn argued that successive paradigms are *incommensurable*.

One advantage of Kuhn's developmental approach to the nature of science is that it draws our attention to the defeasible nature of scientific theories. Even our best theories today may be overthrown down the track and replaced by something we cannot even envisage from our perspective today.

Key Points: According to the Popperian Vision, science proceeds by falsifiable conjecture, and refutation. According to Kuhn, science proceeds by periods of paradigm consensus, punctuated by the occasional radical scientific revolution.

***Question** 8.4.1*

Create your own glossary by writing down definitions of the following terms:

(a) Deductive proof

(b) Experiment

(c) Fact

(d) Hypothesis

(e) Hypothetico-deductive method

(f) Induction

(g) Law

(h) Measurement

(i) Observation

(j) Theory

8.5 Space for additional notes